OUTP-02-30P
IPPP/02/33
DCPT/02/66

Leptogenesis and low-energy phases

Sacha Davidson* **and Alejandro Ibarra**†

* *Dept of Physics, University of Durham, Durham, DH1 3LE, UK*
† *Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, UK*

Abstract

In supersymmetric models, the CP asymmetry produced in the decay of the lightest right-handed neutrino, $\equiv \epsilon$, can be written as a function of weak scale parameters. We introduce a way of separating ϵ into contributions from the various weak-scale phases, and study the contribution of potentially measurable neutrino phases to leptogenesis. We find that the Majorana phase ϕ', which could have observable effects on neutrinoless double beta decay, is important for ϵ unless there are cancellations among phases. If the phase δ can be measured at a neutrino factory, then it contributes significantly to ϵ over much of parameter space.

1 Introduction

CP violation is one of the ingredients [1] required to generate the Baryon Asymmetry of the Universe (BAU) [2]. After the discovery of neutrino oscillations[3,4], leptogenesis [5] stands as one of the most appealing explanations for this asymmetry. Although CP violation has not yet been observed in the leptonic sector, it could perhaps be seen at a neutrino factory, or in neutrinoless double beta decay. It is therefore interesting to investigate whether there is any relation between the CP violation required for leptogenesis and the phases that could be measured at low energies in the neutrino sector.

The seesaw model [6] for neutrino masses is usually analyzed in terms of high-energy parameters, not accessible to experiments, and the resulting predictions are (texture) model-dependent. The above question has been addressed in such an approach [7–9]. Instead, we parametrize the seesaw in terms of weak scale variables [10]. This gives us a model-independent formulation of leptogenesis in terms of low energy inputs, in which we can study the above question.

The aim of this paper is to quantify, in a model independent way, the relation of the CP violation required for leptogenesis to the measurable low-energy phases. We express the CP asymmetry of leptogenesis as a function of real parameters and phases at the weak scale, and then introduce a definition of "phase overlap" between the leptogenesis phase and the individual low energy phases. This definition is not the only possible one, but has linearity properties and is calculable. It is motivated by the notion of vector space, spanned by low energy phases ("basis vectors"), in which the CP asymmetry of leptogenesis is a "vector". The relative importance of a low energy phase for leptogenesis would then be the "inner product" of the leptogenesis "vector" with the relevant "basis vector". We will not be able to construct such a vector space, but it is a useful analogy to keep in mind.

The paper is organized as follows: the next section introduces CP violation in the leptonic sector. Section 3 contains the basic concepts of the supersymmetric see-saw and the generation of the BAU by leptogenesis, from a top-down point of view. In Section 4 we review the procedure to reformulate the see-saw mechanism from a bottom-up perspective. This will allow us to study leptogenesis in terms of low energy data, opening the possibility of relating, in a straight-forward way, the Baryon Asymmetry of the Universe with the CP violation measurable at neutrino factories. In section 5 we develop the general formalism to study quantitatively the above-mentioned relation. In Section 6 and 7 we show the results of our analysis, first for a particular case, and then for a more general case. In section 8 we present a self-contained summary and conclusions. Finally, we include an appendix with the procedure to evaluate numerically the contributions from the low-energy phases to leptogenesis.

2 Flavour and CP violation in the leptonic sector

In the last few years, the Superkamiokande collaboration [4] has provided compelling evidence that neutrinos have mass and oscillate. More recently, the SNO collaboration [11] has confirmed the oscillation hypothesis, and the first neutral current data [12] seem to favour the large angle MSW (LAMSW) solution to the solar neutrino problem [13]. These results, combined with those from a series of other experiments [14], have allowed to measure fairly well the mass splittings and mixing angles relevant for solar and atmospheric neutrino oscillations. In addition to this, other experiments have provided bounds on neutrino parameters from electron antineutrino disappearance (CHOOZ)[15], the non-observation of neutrinoless double beta decay [16], the shape of the tritium beta decay spectrum [17], and different cosmological and astrophysical considerations. However, no evidence has been found so far for CP violation in the leptonic sector.

The search for leptonic CP violation is theoretically motivated by several facts. First, the discovery of CP violation in the leptonic sector could shed some light on the mechanism that generates neutrino masses and perhaps hint at some underlying structure. Secondly, the observation of CP violation in the quark sector, (in the neutral kaon system, ϵ'/ϵ, and in $B \to \psi K_s$), encourages the search for CP violation in the neutrino sector. If there exists a symmetry relating quarks and leptons, these experimental results would point to CP violation also in the leptonic sector. Furthermore, particular models would give definite predictions that could be contrasted in the future. Lastly, and most importantly for the purposes of this paper, CP violation in the leptonic sector could be related to the observed Baryon Asymmetry of the Universe. This is possible in the context of the see-saw mechanism.

On the experimental side, the leptonic version of the CKM phase can be detected by comparing transition probabilities for neutrinos and antineutrinos:

$$A = \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)} \quad . \tag{1}$$

Unfortunately, it is not possible to measure such an asymmetry with the natural sources of neutrinos, i.e. the Sun and pions decaying in the atmosphere, since the "beam" cannot be switched from ν to $\bar{\nu}$. Hence, a lot of effort is being bestowed on the design of a neutrino factory [18,19]: an intense muon source to produce a high-intensity neutrino beam. In the muon storage ring, muons decay to produce muon neutrinos and electron antineutrinos. Whereas a muon neutrino would produce a muon in the detector, the oscillation of an electron antineutrino to a muon antineutrino would produce an antimuon. This antimuon (a "wrong sign" muon) would be a clear signature for oscillation, and $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ could be determined. One of the advantages of a neutrino factory is that the muons in the storage ring can be replaced by antimuons. This makes possible the measurement of $P(\nu_e \to \nu_\mu)$ and hence the CP asymmetry. In practice, detecting CP violation in the neutrino sector is not an easy task [20,19], since the beam has to go through the Earth, that is CP asymmetric. In consequence, the matter effects on the oscillation pattern can obscure the CP violation intrinsic to neutrinos.

If neutrinos have Majorana masses, as predicted by the seesaw mechanism, there are also "Majorana" phases, in addition to the "Dirac" phase that could be detected at a neutrino factory. Neutrinoless double beta decay could be sensitive to these phases (see however [21]). This lepton number violating, but CP conserving, process probes the Majorana neutrino mass matrix element between ν_e and ν_e, which depends on the masses and mixing angles, and also on the Majorana phases. Neutrinoless double beta decay is not observed at the moment. However, experiments which should see a signal, for the currently favored masses and mixing angles (LMA), are being discussed [22].

The see-saw mechanism [6] consists of adding three right-handed neutrinos to the Standard Model (SM) particle content, singlets with respect to the SM gauge group, and coupled to the Higgs doublet through a Yukawa coupling. Then, a Majorana mass term for the right-handed neutrinos is not forbidden by the gauge symmetry, and can be naturally much larger than the scale of electroweak symmetry breaking. These simple assumptions are enough to produce neutrino masses naturally small [1]. Furthermore, if CP is violated in the leptonic sector, the decay of the right-handed neutrinos in the early Universe produces a lepton asymmetry [5,25] that will be eventually reprocessed into a baryon asymmetry by sphalerons [26]. This leptogenesis scenario will be discussed in more detail in section 3.

In supersymmetric models, the seesaw mechanism can induce flavour violating processes involving charged leptons that could be observed in the future [23]. The neutrino Yukawa couplings generate off-diagonal elements in the slepton mass matrix, via renormalization group running. These flavour violating mass terms contribute inside loops

[1] Nevertheless, this minimal model has a serious hierarchy problem: the right-handed neutrinos produce a (large) quadratically divergent radiative correction to the Higgs mass. Therefore, in what follows, we will restrict ourselves to the supersymmetric version of the see-saw mechanism.

to processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. This has been extensively studied from various theoretical [27] and phenomenological [28,29,24] perspectives. The current experimental bound [30] on $\mu \to e\gamma$ imposes some restrictions on the parameter space of the SUSY seesaw. It is anticipated that the sensitivity to $\tau \to \mu\gamma$ and $\mu \to e\gamma$ could improve by as much as three orders of magnitude [31] in forthcoming years. This would provide interesting information about the flavour structure of the SUSY seesaw, irrespective of whether lepton flavour violation is observed or not.

In this paper, we will concentrate on the possible relation of the CP asymmetry in the leptonic sector with the Baryon Asymmetry of the Universe, in the framework of the supersymmetric leptogenesis. We suppose that the BAU is generated in the out-of-equilibrium decay of the *lightest* right-handed neutrino. The CP violation that gives rise to the BAU is not straight-forwardly related to the CP violation that could be observed at low energy. This has been carefully and elegantly discussed, using Jarlskog invariants in [7]. These authors have also studied the relation of the leptogenesis phase to low energy phases in certain classes of models [8]. The goal of this paper is to investigate the interplay between the CP violation at very high energies and at low energies, in a model independent way. We will also comment on the prospects to observe CP violation at a neutrino factory or in neutrinoless double beta decay, in view of the measured BAU, and inversely, what could be inferred about the BAU if CP violation is observed at low energy.

3 The see-saw mechanism and leptogenesis: the top-down approach

The supersymmetric version of the see-saw mechanism has a leptonic superpotential that reads

$$W_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_d + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H_u - \frac{1}{2} {\nu_R^c}^T \mathcal{M} \nu_R^c, \tag{2}$$

where L_i and e_{Ri} ($i = e, \mu, \tau$) are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and H_d (H_u) is the hypercharge $-1/2$ ($+1/2$) Higgs doublet. $\mathbf{Y_e}$ and $\mathbf{Y}_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $\mathcal{M}$ is a 3×3 Majorana mass matrix that does not break the SM gauge symmetry. We do not make any assumptions about the structure of the matrices in eq.(2), but consider the most general case. Then, it can be proved that the number of independent physical parameters is 21: 15 real parameters and 6 phases [33].

It is natural to assume that the overall scale of $\mathcal{M}$, denoted by M, is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian reads

$$\delta\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_d - \frac{1}{2} (\mathbf{Y}_\nu L \cdot H_u)^T \mathcal{M}^{-1} (\mathbf{Y}_\nu L \cdot H_u) + \text{h.c.}. \tag{3}$$

So, after the electroweak symmetry breaking, the left-handed neutrinos acquire a Majorana mass, given by

$$\mathcal{M}_\nu = \mathbf{m_D}^T \mathcal{M}^{-1} \mathbf{m_D}, \tag{4}$$

suppressed with respect to the typical fermion masses by the inverse power of the large scale M.

We will find convenient to work in the flavour basis where the charged-lepton Yukawa matrix, $\mathbf{Y_e}$, and the gauge interactions are flavour-diagonal. In this basis, the neutrino mass matrix, $\mathcal{M}_\nu$, can be diagonalized by the MNS [34] matrix U, defined by

$$U^T m_\nu U = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \equiv D_{m_\nu}, \tag{5}$$

where U is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} , \tag{6}$$

and the m_{ν_i} can be chosen real and positive. Also, we label the masses in such a way that $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. We will assume throughout the paper that the light neutrinos have a hierarchical spectrum. The CP asymmetry required for leptogenesis is suppressed for degenerate neutrinos [35], so we neglect this possibility. U can be written as

$$U = V \cdot \mathrm{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1) \quad , \tag{7}$$

where ϕ and ϕ' are CP violating phases (if different from 0 or π) and V has the ordinary form of the CKM matrix

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \tag{8}$$

It is interesting to note that the neutrino mass matrix, $\mathcal{M}_\nu$, depends on 9 parameters: 6 real parameters and 3 phases. Comparing with the complete theory, we discover that some information has been "lost" in the decoupling process, to be precise, 6 real parameters and three phases. We will return to this important issue later on.

Another remarkable feature of the see-saw mechanism is that it provides a natural framework to generate the baryon asymmetry of the Universe, defined by $\eta_B = (n_B - n_{\bar{B}})/s$, where s is the entropy density. This quantity is strongly constrained by Big Bang Nucleosynthesis to lie in the range $\eta_B \simeq (0.3 - 0.9) \times 10^{-10}$, to successfully reproduce the observed abundances of the light nuclei D, ^{3}He, ^{4}He and ^{7}Li [36]. As was shown by Sakharov, generating a baryon asymmetry requires baryon number violation, C and CP violation, and a deviation from thermal equilibrium. These three conditions are fulfilled in the out-of-equilibrium decay of the right-handed neutrinos and sneutrinos in the early Universe. For conciseness, and since we are concerned only with supersymmetric leptogenesis, in what follows we will use right-handed neutrinos, and the shorthand notation ν_R, to refer both to right-handed neutrinos and right-handed sneutrinos.

Let us briefly review the mechanism of generation of the BAU through leptogenesis [5,25]. At the end of inflation, a certain number density of right-handed neutrinos, n_{ν_R}, is produced, that depends on the cosmological scenario. These right-handed neutrinos decay, with a decay rate that reads, at tree level,

$$\Gamma_{D_i} = \Gamma(\nu_{R_i} \to \ell_i H_u) + \Gamma(\nu_{R_i} \to \tilde{L}_i \tilde{h}_u) = \frac{1}{8\pi} (\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger)_{ii} M_i. \tag{9}$$

The out of equilibrium decay of a right-handed neutrino ν_{R_i} creates a lepton asymmetry given by

$$\eta_L = \frac{n_\ell - n_{\bar{\ell}}}{s} = \frac{n_{\nu_R} + n_{\tilde{\nu}_R}}{s} \, \epsilon_i \, \kappa. \tag{10}$$

The value of $(n_{\nu_R} + n_{\tilde{\nu}_R})/s$ depends on the particular mechanism to generate the right-handed (s)neutrinos. On the other hand, the CP-violating parameter

$$\epsilon_i = \frac{\Gamma_{D_i} - \bar{\Gamma}_{D_i}}{\Gamma_{D_i} + \bar{\Gamma}_{D_i}}, \tag{11}$$

where $\bar{\Gamma}_{D_i}$ is the CP conjugated version of Γ_{D_i}, is determined by the particle physics model that gives the masses and couplings of the ν_R. Finally, κ is the fraction of the produced asymmetry that survives washout by lepton number violating interactions after ν_R decay. To ensure $\kappa \sim 1$, lepton number violating interactions (decays, inverse decays and scatterings) must be out of equilibrium when the right-handed neutrinos decay. In the case of the lightest right-handed neutrino ν_{R_1}, this corresponds approximately to $\Gamma_{D_1} < H|_{T \simeq M_1}$, where H is the Hubble parameter at the temperature T, and can be expressed in terms of an effective light neutrino mass [25,37], $\widetilde{m}_1$, as

$$\widetilde{m}_1 = \frac{8\pi \langle H_u^0 \rangle^2}{M_1^2} \Gamma_{D_1} = (\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger)_{11} \frac{\langle H_u^0 \rangle^2}{M_1} \lesssim 5 \times 10^{-3} \mathrm{eV}. \tag{12}$$

This requirement has been carefully studied [25,37,45]; the precise numerical bound on $\widetilde{m}_1$ depends on M_1, and can be found in [37].

The last step is the transformation of the lepton asymmetry into a baryon asymmetry by non-perturbative B+L violating (sphaleron) processes [26], giving

$$\eta_B = \frac{C}{C-1} \eta_L, \tag{13}$$

where C is a number $\mathcal{O}(1)$, that in the Minimal Supersymmetric Standard Model takes the value $C = 8/23$.

In this paper, we assume that a sufficient number of ν_R were produced — thermally, or in the decay of the inflaton, or as a scalar condensate of $\tilde{\nu}_R$s, or by some other mechanism. We will concentrate on the step of leptogenesis that is most directly related to neutrino physics, namely the generation of a CP asymmetry, ϵ, in the decay of the right handed neutrinos. It is convenient to work in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$\mathcal{M} = \mathrm{diag}(M_1, M_2, M_3), \tag{14}$$

with M_i real and $0 \leq M_1 < M_2 < M_3$. In this basis, the CP asymmetry produced in the decay of ν_{R_i} reads

$$\epsilon_i \simeq -\frac{1}{8\pi}\frac{1}{[\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger]_{ii}} \sum_j \mathrm{Im}\left\{[\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger]^2_{ij}\right\} f\left(\frac{M_j^2}{M_i^2}\right), \tag{15}$$

where [39]

$$f(x) = \sqrt{x}\left(\frac{2}{x-1} + \ln\left[\frac{1+x}{x}\right]\right). \tag{16}$$

Here, we will assume that the masses of the right-handed neutrinos are hierarchical. We also assume that the lepton asymmetry is generated in the decay of the lightest right-handed neutrino. This second assumption is critical; if the asymmetry was generated by the decay of ν_{R_2} or ν_{R_3}, it would depend on a different combination of phases. This assumption is also dubious if the ν_R are produced thermally, because the ν_{R_1} mass, M_1, is severely constrained in SUSY models. To get a large enough baryon asymmetry, $M_1 > 10^8$ GeV is required [35,38], but M_1 must be less than or of order the reheat temperature T_{reh}. To avoid overproducing gravitons in the early Universe, T_{reh} is required to be $\lesssim 10^9 - 10^{10}$ GeV [40].

With these approximations, the CP asymmetry is

$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{[\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger]_{11}} \sum_j \mathrm{Im}\left\{[\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger]^2_{1j}\right\} \left(\frac{M_1}{M_j}\right) \tag{17}$$

$$= -\frac{3}{8\pi \langle H_u^0\rangle^2}\frac{M_1}{[\mathbf{Y}_\nu \mathbf{Y}_\nu{}^\dagger]_{11}} \mathrm{Im}\left\{[\mathbf{Y}_\nu \mathcal{M}_\nu{}^\dagger \mathbf{Y}_\nu{}^T]_{11}\right\}. \tag{18}$$

The CP asymmetry depends on quantities that appear in the superpotential of the complete theory, eq.(2), and that are not directly measurable with experiments. However, these quantities can be related to neutrino and sneutrino parameters, as we will discuss in the next section. One of the goals of this paper is to implement in an explicit way these constraints on the CP asymmetry, eq.(18).

4 The see-saw mechanism and leptogenesis: the bottom-up approach

Our starting point will be the procedure presented in [10]. In the basis defined in section 3, where the charged lepton mass matrix and the right-handed Majorana mass matrix are diagonal, the neutrino Yukawa coupling must be necessarily non-diagonal. However, it can be diagonalized by two unitary transformations:

$$\mathbf{Y}_\nu = V_R^\dagger D_Y V_L. \tag{19}$$

It is clear that the CP asymmetry depends just on V_R and $D_{\mathcal{M}}$. These quantities are related to the physics of the right-handed neutrinos and are not directly testable by experiments, since they are related to very high energy physics. However, there is a reminiscence of V_R and $D_{\mathcal{M}}$ in the low energy neutrino mass matrix that can be exploited to obtain information about the high-energy physics from the neutrino data. Substituting eq.(19) in the see-saw formula, $\mathcal{M}_\nu = \mathbf{m_D}^T \mathcal{M}^{-1} \mathbf{m_D}$, one obtains

$$D_Y^{-1} V_L^* \frac{\mathcal{M}_\nu}{\langle H_u^0 \rangle^2} V_L^\dagger D_Y^{-1} = V_R^* D_\mathcal{M}^{-1} V_R^\dagger \equiv \mathcal{M}^{-1}. \tag{20}$$

From this equation we can solve for V_R and $D_\mathcal{M}$ in terms of $\mathcal{M}_\nu$, D_Y and V_L. $\mathcal{M}_\nu$ is constrained by neutrino experiments, whereas $D_Y = \mathrm{diag}(y_1, y_2, y_3)$ and V_L are unknown parameters at this stage. We choose a parametrization of the unitary matrix V_L such that

$$V_L = \begin{pmatrix} c_{13}^L c_{12}^L & c_{13}^L s_{12}^L e^{-i\varphi_{12}} & s_{13}^L e^{-i\varphi_{13}} \\ -c_{23}^L s_{12}^L e^{i\varphi_{12}} - s_{23}^L s_{13}^L c_{12}^L e^{i(\varphi_{13}-\varphi_{23})} & c_{23}^L c_{12}^L - s_{23}^L s_{13}^L s_{12}^L e^{-i(\varphi_{12}-\varphi_{13}+\varphi_{23})} & s_{23}^L c_{13}^L e^{-i\varphi_{23}} \\ s_{23}^L s_{12}^L e^{i(\varphi_{12}+\varphi_{23})} - c_{23}^L s_{13}^L c_{12}^L e^{i\varphi_{13}} & -s_{23}^L c_{12}^L e^{i\varphi_{23}} - c_{23}^L s_{13}^L s_{12}^L e^{i(\varphi_{13}-\varphi_{12})} & c_{23}^L c_{13}^L \end{pmatrix}, \tag{21}$$

where $c_{ij}^L = \cos\theta_{ij}^L$ and $s_{ij}^L = \sin\theta_{ij}^L$, being θ_{ij}^L the angles in the V_L matrix.

Interestingly enough, in certain scenarios, the parameters D_Y and V_L can be constrained experimentally. For example, in a scenario of minimal SUGRA, with just the MSSM+3ν_Rs below the GUT scale, D_Y and V_L can in principle be extracted from the radiative corrections to the left-handed slepton mass matrix, since the corresponding RGE depends on the combination $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L$. To be more precise, at low energies the left-handed slepton mass matrix reads, in the leading-log approximation

$$\left(m^2_{\tilde{\ell},\tilde{\nu}}\right)_{ij} \simeq (\text{diagonal part})_{\tilde{\ell},\tilde{\nu}} + \frac{1}{8\pi^2}(3m_0^2 + A_0^2)\mathbf{Y}^\dagger_{\nu_{ik}} \mathbf{Y}_{\nu_{kj}} \log \frac{M_k}{M_{GUT}} \ . \tag{22}$$

The off-diagonal terms in $m^2_{\tilde{\ell},\tilde{\nu}}$ manifest themselves in processes like $\mu \to e\gamma$ or $\tau \to \mu\gamma$, that could be observed in the near future [2]. In addition to this, at tree level the three sneutrino masses are degenerate. However, radiative corrections induce a non-universality among the masses that could perhaps be measured experimentally [42]. All these measurements could be used to disentangle some information about the neutrino Yukawa matrix and the right handed masses from radiative corrections. A more detailed discussion of obtaining information about the complete theory from low energy data can be found in [10]; see [43] for a recent analysis of $\ell_j \to \ell_i\gamma$ in this approach.

The V_L, D_Y low-energy parametrization has several advantages. If we treat the 9 parameters of the neutrino mass matrix as "known", there are 9 remaining unknown variables in the seesaw: three phases and six real numbers. Possible parametrizations of these unknowns are D_Y and V_L, $D_\mathcal{M}$ and the orthogonal complex matrix R [24], or as in [43]. The angles and phases of V_L are related in a simple way to the lepton flavour violating slepton mass matrix entries. These off-diagonal (in the charged lepton mass eigenstate basis) entries are currently constrained and could possibly be determined by radiative lepton decays $\ell_j \to \ell_i\gamma$. The eigenvalues of D_Y are more difficult to determine

[2] As a matter of fact, the upper bounds on the off-diagonal entries of $\mathbf{Y}^\dagger_{\nu_{ik}} \mathbf{Y}_{\nu_{kj}} \log \frac{M_X}{M_k}$ in the scenario of mSUGRA with the MSSM+3ν_Rs below the GUT scale apply for a wide class of models, since one does not expect cancellations among the different terms in the RGEs, or with the off-diagonal elements of the tree-level slepton mass matrix.

experimentally. However, we do measure the Yukawa matrix eigenvalues for the quarks and charged leptons, so we can make theoretical guesses of the Y_ν eigenvalues with more confidence than *e.g.* guessing the ν_R Majorana masses.

It is convenient for our leptogenesis analysis to parametrize the sneutrino mass matrix with D_Y and V_L. It would be more correct to express the lepton asymmetry in terms of the magnitude and phases of slepton mass matrix elements [3]. Alternatively, there is an intermediate parametrization, which can be useful for analytic estimates. The parameters we use, V_L and D_Y, determine $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ rather than $\mathbf{Y}^\dagger_{\nu_{ik}} \mathbf{Y}_{\nu_{kj}} \log \frac{M_k}{M_{GUT}}$, which is the expression that appears at leading log. It is easy, though, to relate V_L and D_Y to $\mathbf{Y}^\dagger_{\nu_{ik}} \mathbf{Y}_{\nu_{kj}} \log \frac{M_k}{M_{GUT}}$. Noting that

$$\mathbf{Y}^\dagger_{\nu ik} \log \frac{M_k}{M_{GUT}} \mathbf{Y}_{\nu_{kj}} = (\widetilde{\mathbf{Y}}_\nu^\dagger \widetilde{\mathbf{Y}}_\nu)_{ij} = (\widetilde{V}_L^+ \widetilde{D}_Y^2 \widetilde{V}_L)_{ij} \tag{23}$$

$$\frac{\mathcal{M}_{\nu_{ij}}}{\langle H_u^0 \rangle^2} = \widetilde{\mathbf{Y}}^T_{\nu_{ik}} \frac{1}{\widetilde{M}_k} \widetilde{\mathbf{Y}}_{\nu_{kj}},$$

where $\widetilde{\mathbf{Y}}_{ki} = \mathbf{Y}_{ki} \sqrt{\log \frac{M_k}{M_{GUT}}}$ and $\widetilde{M}_k = M_k \log \frac{M_k}{M_{GUT}}$, it is possible to rewrite eq.(20) but using tilded parameters. So, one could parametrize the see-saw mechanism with the neutrino mass matrix, $\mathcal{M}_\nu$, and $\widetilde{V}_L$, $\widetilde{D}_Y$, that are directly related to the leading-log approximate solution of the left-handed slepton RGEs. Also, from the definitions, it is straightforward to relate V_L and D_Y with their tilded-counterparts. However, since SUSY has not yet been discovered, we use V_L and D_Y, with the knowledge that we can calculate $[m_{\tilde{\nu}}^2]$ from these parameters. This choice will be important when we discuss phase overlaps.

We turn now to expressing the CP asymmetry in terms of neutrino masses, the MNS matrix, and other unknown parameters encoded in D_Y and V_L. We can make an analytic approximation indicating the dependence of the CP asymmetry ϵ on our low energy parameters. To derive these estimates, we first assume $M_3 \gg M_1$ and $y_1 \ll y_2, y_3$. Then we assume that $[\mathcal{M}^{-1\dagger}\mathcal{M}^{-1}]_{11}$ is the largest element of $\mathcal{M}^{-1\dagger}\mathcal{M}^{-1}$, in the basis where Y_ν is diagonal. As we will see, this is usually reasonable.

If a matrix Λ has a zero eigenvalue, then the remaining two eigenvalues are

$$\lambda_1, \lambda_2 = \frac{1}{2} \left\{ \mathrm{Tr}\Lambda \pm \sqrt{(\mathrm{Tr}\Lambda)^2 - 4\left(\Lambda_{11} \mathrm{tr}\Lambda + \det\Lambda - [\Lambda_{12}]^2 - [\Lambda_{13}]^2\right)} \right\} \quad , \tag{24}$$

where Tr is the trace of the 3-d matrix, and tr and det are defined on the 2-3 subspace. In the limit where $M_3 \to \infty$, this formula can be applied to the hermitian matrix $\mathcal{M}^{-1\dagger}\mathcal{M}^{-1}$:

$$\mathcal{M}^{-1\dagger}\mathcal{M}^{-1} = D_Y^{-1} V_L \kappa^\dagger V_L^T D_Y^{-2} V_L^* \kappa V_L^\dagger D_Y^{-1} \equiv \frac{\Lambda}{y_1^4} \quad . \tag{25}$$

To obtain simple expressions, we would like to expand eq. (24) in small dimensionless parameters. So to avoid confusion, we scale a factor y_1^4 out of $\mathcal{M}^{-1}\mathcal{M}$.

The largest eigenvalue of Λ will be of order m_ν^2, as can be seen by defining $\eta \equiv y_1 D_Y^{-1} = diag\{\eta_1, \eta_2, \eta_3\}$ and

$$\Delta = V_L^* \kappa V_L^\dagger = V_L^* U^* D_{m_\nu} U^\dagger V_L^\dagger \equiv W^* D_{m_\nu} W^\dagger \quad , \tag{26}$$

[3] We will follow this approach in a subsequent publication [44].

which gives

$$\Lambda = \eta \Delta^\dagger \eta^2 \Delta \eta \quad . \tag{27}$$

We take $y_3 = 1$. The matrix $W = V_L U$ is the rotation from the basis where the ν_L masses are diagonal to the basis where the neutrino Yukawa matrix $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is diagonal.

The dominant contributions to the matrix elements of Λ can be calculated as an expansion in η_2 and η_3. Only Λ_{11} is zeroth order in η_2 and η_3, so generically $\Lambda_{11} \gg \Lambda_{22}, \Lambda_{33}$. Then from eq. (24), the lightest RH neutrino has mass

$$|M_1|^2 \simeq \frac{y_1^4}{\Lambda_{11}} \quad , \tag{28}$$

and the associated eigenvector will be

$$\vec{v}_1 \simeq \begin{pmatrix} \Lambda_{11} \\ \Lambda_{21} \\ \Lambda_{31} \end{pmatrix} \times \frac{1}{\Lambda_{11}} = \begin{pmatrix} \Delta_{11}^* \\ \eta_2 \Delta_{12}^* \\ \eta_3 \Delta_{13}^* \end{pmatrix} \times \frac{1}{\Delta_{11}^*} \quad . \tag{29}$$

We can use this eigenvector to evaluate eq. (18), and find

$$\epsilon \simeq -\frac{3 y_1^2 \Lambda_{11}^2}{8\pi [\Lambda D_Y^2 \Lambda]_{11}} \mathrm{Im} \left\{ \frac{[\Lambda D_Y \Delta^\dagger D_Y \Lambda^T]_{11}}{[\Lambda \eta \Delta^\dagger \eta \Lambda^T]_{11}} \right\} = \frac{3 y_1^2}{8\pi \sum_j |W_{1j}|^2 m_{\nu_j}^2} \mathrm{Im} \left\{ \frac{\sum_k W_{1k}^2 m_{\nu_k}^3}{\sum_n W_{1n}^2 m_{\nu_n}} \right\} \quad , \tag{30}$$

where we have dropped terms of order η_2 and η_3, and recall that W is the rotation from the basis where the ν_L masses are diagonal to the basis where $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is diagonal.

It is important to notice that the CP asymmetry depends only on the first row of the matrix W, that in turn depends only on the first row of V_L. In the parametrization that we have chosen for V_L, eq.(21), the first row depends on two angles and two phases. Therefore, at the end of the day, the CP asymmetry depends on the neutrino mass matrix and five unknown parameters: y_1, two angles and two phases. Note that for generic Δ, the order of magnitude of ϵ is fixed by y_1^2. For the GUT-inspired value $y_1 \simeq m_u/m_t \sim 10^{-4}$, $\epsilon \sim 10^{-9}$ unless there is some amplification in $\mathrm{Im}\{[\Delta\Delta^\dagger\Delta]_{11}\Delta_{11}^*\}/([\Delta\Delta^\dagger]_{11}|\Delta_{11}|^2)$.

5 Phases for leptogenesis

From the previous discussion, we find that in the parametrization we have chosen, the CP asymmetry depends on five phases, namely the phases in the MNS matrix, δ, ϕ and ϕ', and the phases in the first row of the V_L matrix, φ_{12} and φ_{13}. In this section we would like to study the relative importance of these phases on the CP asymmetry, and whether any of them could be considered as the "leptogenesis phase", i.e. the phase that is fully responsible of the CP asymmetry.

To this end, we first introduce a definition of "overlap" between the "leptogenesis phase" and the low energy phases. At the end of the section, we will discuss issues raised by our definition.

We define the contribution to the CP asymmetry from a phase α (this is *not* quite what we call a phase overlap) such that the total CP asymmetry is the sum of the different contributions:

$$\epsilon = \epsilon_\delta + \epsilon_\phi + \epsilon_{\phi'} + \epsilon_{\varphi_{12}} + \epsilon_{\varphi_{13}} \quad . \tag{31}$$

To obtain a decomposition of the CP asymmetry in this way, and give a more precise definition of the different contributions, we Fourier expand the CP asymmetry:

$$\epsilon = \sum_{j,k,l,m,n} A_{jklmn} \sin(j\delta + k\phi + l\phi' + m\varphi_{12} + n\varphi_{13}) \quad . \tag{32}$$

This summation can be split in

$$\epsilon = \sum_{\alpha} C_\alpha + \sum_{\alpha<\beta} C_{\alpha\beta} + \sum_{\alpha<\beta<\gamma} C_{\alpha\beta\gamma} + \sum_{\alpha<\beta<\gamma<\rho} C_{\alpha\beta\gamma\rho} + \sum_{\alpha<\beta<\gamma<\rho<\sigma} C_{\alpha\beta\gamma\rho\sigma} \quad , \tag{33}$$

with $\{\alpha, \beta, \gamma, \rho, \sigma\}$ elements of the ordered set $\{\delta, \phi, \phi', \varphi_{12}, \varphi_{13}\}$. Note that the subindices of the C's are ordered, so $\delta < \phi < \phi' < \varphi_{12} < \varphi_{13}$. (to avoid double counting, only $C_{\delta\phi}$ exists, and $C_{\phi\delta}$ does not.) Some of the terms in the summation are:

$$C_\delta = \sum_{j\neq 0} A_{j0000} \sin(j\delta) \tag{34}$$

$$C_{\delta\phi} = \sum_{j\neq 0, k\neq 0} A_{jk000} \sin(j\delta + k\phi) \tag{35}$$

$$C_{\delta\phi\phi'} = \sum_{j\neq 0, k\neq 0, l\neq 0} A_{jkl00} \sin(j\delta + k\phi + l\phi') \tag{36}$$

and so on.

We can now rewrite the summation eq.(33) in a way that resembles eq.(31). It is clear that C_δ is a contribution from δ to the CP asymmetry, so it must be one of the terms in ϵ_δ. On the other hand, $C_{\delta\phi}$ is a contribution from δ, but also from ϕ, and it is not possible to conclude whether it is a contribution from δ or from ϕ. So, we will say that $C_{\delta\phi}$ contributes in $C_{\delta\phi}/2$ to ϵ_δ and in $C_{\delta\phi}/2$ to ϵ_ϕ. This rationale can be applied to the rest of the terms in the expansion eq.(33) to finally obtain

$$\epsilon_\delta = C_\delta + \frac{1}{2}\sum_{\beta} C_{\delta\beta} + \frac{1}{3}\sum_{\beta<\gamma} C_{\delta\beta\gamma} + \frac{1}{4}\sum_{\beta<\gamma<\rho} C_{\delta\beta\gamma\rho} + \frac{1}{5}\sum_{\beta<\gamma<\rho<\sigma} C_{\delta\beta\gamma\rho\sigma} \quad , \tag{37}$$

and similarly for ϵ_ϕ, $\epsilon_{\phi'}$, $\epsilon_{\varphi_{12}}$ and $\epsilon_{\varphi_{13}}$. It can be checked that with this decomposition, eq.(31) holds.

In this analysis, we are only concerned with the relative contributions of the different phases to the CP asymmetry, and not with the overall magnitude of the CP asymmetry itself (that is essentially determined by the unknown parameter y_1). So, we normalize the different contributions to 1, and define "phase overlap" as the normalized contribution from a phase to the CP asymmetry. This quantity measures the relative importance of that phase for the CP asymmetry compared to the rest of the phases. For instance, the "overlap" of δ with the leptogenesis phase is:

$$O_\delta = \frac{|\epsilon_\delta|}{\sqrt{\sum_\alpha \epsilon_\alpha^2}} \quad , \tag{38}$$

which satisfies $\sum O_\alpha^2 = 1$ and $0 \leq O_\alpha \leq 1$. Had we chosen a linear normalization, i.e. $O_\alpha = \epsilon_\alpha/\epsilon$, in some regions of the parameter space $|O_\alpha|$ could be larger than one, due to cancellations among the different ϵ_α's. So, we prefer to use a quadratic normalization, that satisfies $0 \leq O_\alpha \leq 1$ to better represent the fractional contribution of the phase α to ϵ.

The overlap defined in eq.(38) measures the importance of δ for ϵ, provided that the phases in the expansion are independent and "orthogonal". In any parametrization of the seesaw, six phases are required, so independence is automatic. The importance of "orthogonality" can be understood by analogy with linear algebra, where a vector can be uniquely decomposed in its components along a given orthonormal basis. Similarly, the phases of our parametrization must be "orthogonal", as well as independent, to have a unique definition of the fraction of ϵ due to δ. However, a mathematical definition of "orthogonality" is difficult, perhaps impossible, because we do not have an inner product between phases. So we opt for a physical notion: we assume as "orthogonal" the so-called "physical" phases, the phases that could be measured at low energy — in practice, or in principle in the best of all physicists worlds. The situation is analogous to the analyses of the Constrained MSSM, whose parameter space is spanned by the universal scalar mass (m_0), gaugino mass (M_0) and trilinear term (A_0), $\tan\beta$ and signμ. Since we ignore the particular mechanism of SUSY breaking, that moreover is not directly testable by experiments, we regard these parameters as independent and "orthogonal", and scan the parameter space looking for predictions that are independent of the details of the SUSY breaking. Obviously, in a top-down approach to supersymmetry, where a particular SUSY breaking scenario is assumed, there exists relations among these parameters (for example, in a string scenario with dilaton dominated SUSY breaking, $M_0 = \pm\sqrt{3}m_0$, $A_0 = -M_0$). Similarly, in a top-down approach to the see-saw mechanism, where a neutrino Yukawa texture is assumed, our low-energy phases could not look "orthogonal", but we can only know about these relations if we know the details of the neutrino Yukawa texture, that are not accessible to experiments (as is the SUSY breaking physics). So, following the same spirit as in the more familiar analyses of the Constrained MSSM, we forget about the high-energy physics and span the parameter space by those quantities that are most closely related to experiments, assuming that they are "orthogonal".

Notice that the choice of low energy phases is important — had we parametrized with the phases of U and W, then from eq. (30), ϵ depends only on the phases of W_{1i} and is independent of the MNS phases. Similarly, if the seesaw is parametrized using U and the complex orthogonal matrix R, The MNS matrix cancels out of the equation for ϵ, and the δ dependence of ϵ is buried in R.

The measurable phases of the slepton sector are those of the slepton mass matrices, so we should expand ϵ on δ, ϕ', ϕ, and the phases of $[\tilde{m}^2]_{ij}$. However, we find it convenient to parametrize the seesaw in terms of V_L and D_Y, rather than $[\tilde{m}_L^2]$, as discussed in section 4. The $[\tilde{m}^2]_{ij}$ are therefore functions of the real angles of V_L, as well as the phases of V_L, so φ_{12} and φ_{13} are not quite the correct physical phases. We expect this choice to have little effect on the relative importance of δ and ϕ' for leptogenesis. V_L is closely related to the slepton mass matrix, and in the limit that the real angles in V_L are small, the phases of $\tilde{m}_{\tilde{\nu}}^2$ are those of V_L (or $\tilde{V}_L$) in leading log.

To avoid possible confusion, observe that our notion of "leptogenesis phase" differs from the one introduced in [38], who write $\epsilon = \epsilon_{max} \sin\delta$ where ϵ_{max} is the upper bound on ϵ. The asymmetry is the imaginary part of a complex number $\epsilon \equiv \mathrm{Im}\{|\epsilon_c| e^{i\rho}\} = |\epsilon_c| \sin\rho$, so we interpret the "leptogenesis phase" to be ρ.

The definition, eq. (38), of the fraction of ϵ that is due to δ, depends on eight unknowns: three real angles and the five phases. We assume that the real angles could be measured, so we present results for different fixed values of the angles. We take random values of the phases, linearly distributed between 0 and 2π, and make scatter plots of the overlaps $\{O_\alpha\}$. If most of the points are distributed at $|O_\alpha|^2 > .3$, we conclude that the phase α contributes significantly to ϵ (α here represents any phase among $\{\delta, \phi', \phi, \varphi_{12}, \varphi_{13}\}$). Notice however that this is a statistical statement, based on choosing all the phases randomly and large.

6 The case $V_L = 1$

In this section we particularize the previous study to the case $V_L = 1$. In this case, there is no flavour or CP violation induced radiatively by right-handed neutrinos in the slepton mass matrices. Since we have fixed the V_L matrix, the number of unknown parameters is reduced, and the CP asymmetry depends just on the neutrino mass matrix and y_1, the lightest eigenvalue of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$.

When $V_L = 1$, only the phases in the MNS matrix, δ, ϕ and ϕ' are relevant, hence the analysis of the previous section is greatly simplified. The CP asymmetry can be written as the sum of the contributions from the phases δ, ϕ and ϕ',

$$\epsilon = \epsilon_\delta + \epsilon_\phi + \epsilon_{\phi'} \quad . \tag{39}$$

As in the previous section, we Fourier expand ϵ, yielding

$$\epsilon = \sum_{j,k,l} A_{jkl} \sin(j\delta + k\phi + l\phi') = C_\delta + C_\phi + C_{\phi'} + C_{\delta\phi} + C_{\delta\phi'} + C_{\phi\phi'} + C_{\delta\phi\phi'} \quad , \tag{40}$$

with C_α, $C_{\alpha\beta}$ and $C_{\delta\phi\phi'}$ as in eqs.(34)-(36). Then, the contribution from the phases δ, ϕ and ϕ' to the CP asymmetry read

$$\begin{aligned} \epsilon_\delta &= C_\delta + \frac{1}{2}(C_{\delta\phi} + C_{\delta\phi'}) + \frac{1}{3} C_{\delta\phi\phi'} \\ \epsilon_\phi &= C_\phi + \frac{1}{2}(C_{\delta\phi} + C_{\phi\phi'}) + \frac{1}{3} C_{\delta\phi\phi'} \\ \epsilon_{\phi'} &= C_{\phi'} + \frac{1}{2}(C_{\delta\phi'} + C_{\phi\phi'}) + \frac{1}{3} C_{\delta\phi\phi'} \quad . \end{aligned} \tag{41}$$

As before, and since we are only concerned with the relative contributions from the different phases to ϵ and not with the overall magnitude, we define the "phase overlaps" as:

$$O_\alpha = \frac{|\epsilon_\alpha|}{\sqrt{\epsilon_\delta^2 + \epsilon_\phi^2 + \epsilon_{\phi'}^2}}, \tag{42}$$

where $\alpha = \delta, \phi, \phi'$. With these definitions, the following identity holds:

$$(O_\delta)^2 + (O_\phi)^2 + (O_{\phi'})^2 = 1 \quad . \tag{43}$$

In Figure 1 we show the numerical results for different CHOOZ angles. We show the results for the LAMSW solution to the solar neutrino problem and the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$. Each point corresponds to a random value of the phases δ, ϕ and ϕ' between 0 and 2π. In view of eq.(43), we find convenient to present the results on a triangular plot, where the distance to the sides of the triangle corresponds to the "phase overlaps" squared, defined in eq.(42) (see upper left plot).

When the CHOOZ angle is close to the experimental bound (upper right plot) over most of the parameter space the relevant phases are δ and ϕ', and their contributions are approximately equal. In this case, the phase ϕ is essentially irrelevant, except for a few points that correspond to $2\delta - \phi' \simeq 0, \pi$. On the other hand, when the CHOOZ angle is moderately small (lower left plot), we find points scattered over the whole triangle: the three phases are relevant in this case. One can also see from the figure that the points seem to follow a circular pattern. We will come back to this issue later on. Finally, when the CHOOZ angle is very small (lower right plot), the relevant phases are ϕ and ϕ', except for the points for which $\phi - \phi' \simeq 0, \pi$, where the phase δ becomes relevant. A neutrino factory is expected to be sensitive to $\sin\theta_{13} \gtrsim 10^{-4}$ [19] and to be able to see CP violation for phases of order 1 is $\sin\theta_{13} \gtrsim .01$ [20].

These plots can be understood analytically using the approximation for the CP asymmetry, eq.(30). When $V_L = 1$, the CP asymmetry has a fairly simple expression in terms of low energy neutrino data and y_1, the lightest eigenvalue of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$:

$$\epsilon \simeq -\frac{3y_1^2}{8\pi D}\,\mathrm{Im}\left\{\frac{m_{\nu_1}^3\; c_{13}^2 c_{12}^2\; e^{i\phi} + m_{\nu_2}^3\; c_{13}^2 s_{12}^2\; e^{i\phi'} + m_{\nu_3}^3\; s_{13}^2\; e^{2i\delta}}{m_{\nu_1}\; c_{13}^2 c_{12}^2\; e^{i\phi} + m_{\nu_2}\; c_{13}^2 s_{12}^2\; e^{i\phi'} + m_{\nu_3}\; s_{13}^2\; e^{2i\delta}}\right\} \quad , \tag{44}$$

where $D = m_{\nu_1}^2\; c_{13}^2 c_{12}^2 + m_{\nu_2}^2\; c_{13}^2 s_{12}^2 + m_{\nu_3}^2\; s_{13}^2$. For the mass hierarchy and the ranges of CHOOZ angles that we are using, it turns out that $m_{\nu_2} \gg m_{\nu_1},\ m_{\nu_3} s_{13}^2$, so we can expand the denominator in ϵ. Approximating $\theta_{12} \sim \pi/4$, the result is:

$$\epsilon \simeq -\frac{3y_1^2}{4\pi}\left\{\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 2 s_{13}^2 \sin(2\delta - \phi') - \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi')\right\} \quad . \tag{45}$$

When the CHOOZ angle is much larger than $\sqrt{m_{\nu_1} m_{\nu_2}^2 / m_{\nu_3}^3}$ the first term in eq.(45) dominates, unless $2\delta - \phi'$ is close to 0 or π. This condition is satisfied in particular when the lightest neutrino is very light, which is an interesting physical possibility. For the mass hierarchy that we have chosen, the condition above reads $s_{13} \gg 0.01$, which

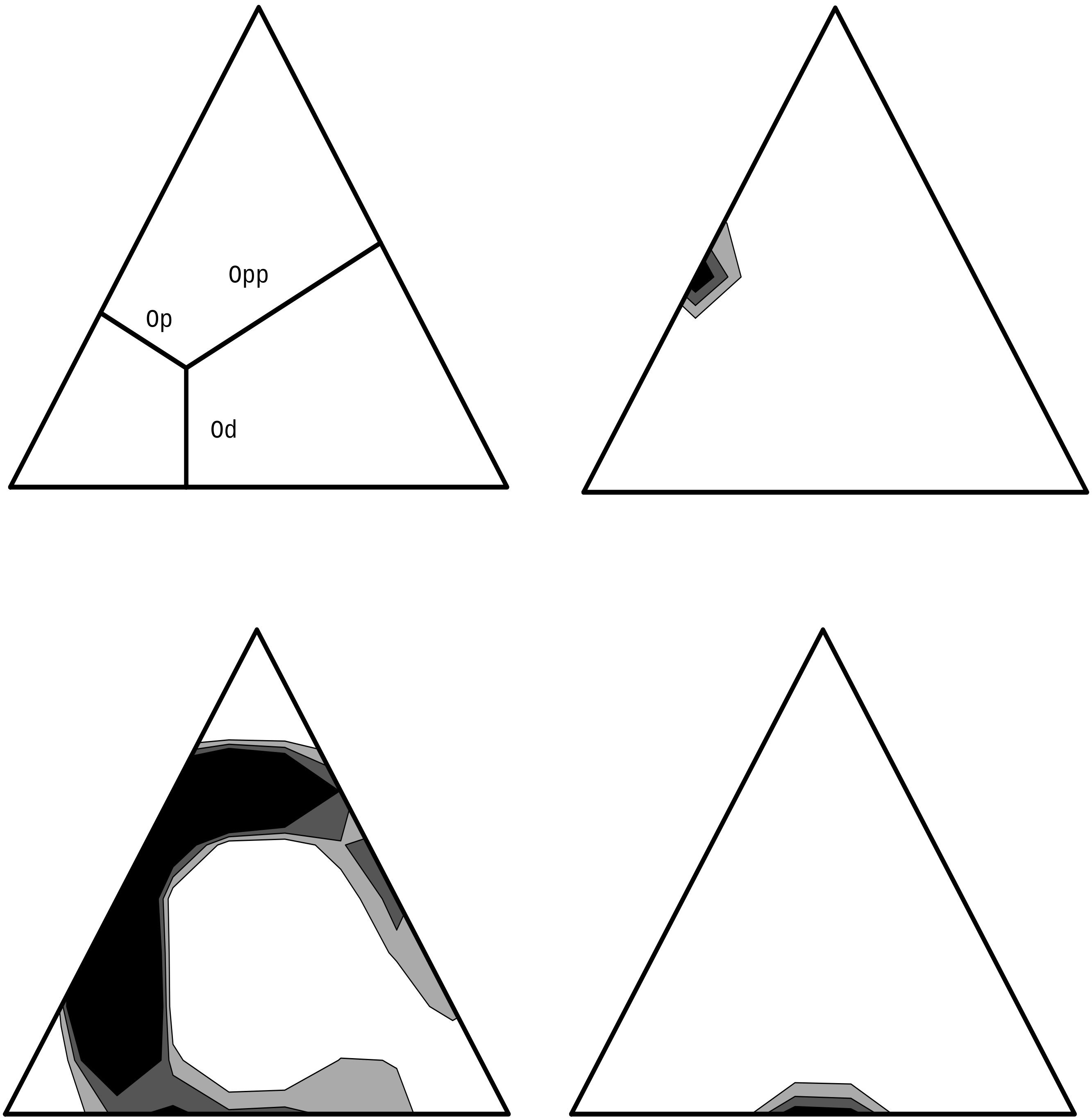

Figure 1: "Phase overlaps" for the case $V_L = 1$, i.e. $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ diagonal. The upper left plot indicates the meaning of the distances to the different sides of the triangle. The rest of the triangles show density plots of the "phase overlaps" for random values of the phases and different CHOOZ angles: 0.1 (upper right), 0.01 (lower left) and 0.001 (lower right). The darkest regions correspond to the largest density of points.

is satisfied when the CHOOZ angle is close the present experimental limit. Recall that $\theta_{13} \gtrsim .01$ is required to detect δ at a neutrino factory, so this limit would hold if CP violation is observed at the neutrino factory. The CP asymmetry in this case can be approximated by

$$\epsilon \simeq -\frac{3y_1^2}{2\pi}\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta - \phi'), \tag{46}$$

that does not depend on ϕ; only on δ and ϕ'. Furthermore, the dependence is such that one cannot conclude whether the "leptogenesis phase" is δ or ϕ'. Instead, in this limit, the "leptogenesis phase" is the combination $2\delta - \phi'$. Comparing eq.(46) with the Fourier expansion, eq.(40), it follows that for most points, $\epsilon \simeq C_{\delta\phi'}$. Hence $\epsilon_\phi \simeq 0$, $\epsilon_\delta \simeq \epsilon_{\phi'} \simeq C_{\delta\phi'}/2$. ($\epsilon_\delta \simeq \epsilon_{\phi'}$ is a consequence of the fact that ϵ depends on a combination of δ and ϕ'.) Consequently, most points in the scatter plot, Fig. 1, upper right, are concentrated in the middle of the side corresponding to $O_\phi = 0$.

On the other hand, when the CHOOZ angle is very small, it is the second term in eq.(45) the one that dominates, as long as $(\phi - \phi')$ is different from 0 or π. In this limit,

$$\epsilon \simeq \frac{3y_1^2}{4\pi}\,\frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi') \quad . \tag{47}$$

The CP asymmetry only depends on ϕ and ϕ', and the "leptogenesis phase" is $\phi - \phi'$. As before, comparing eq.(47) with the expansion eq.(40), we conclude that for most points, $\epsilon \simeq C_{\phi\phi'}$. Hence, $\epsilon_\delta \simeq 0$, $\epsilon_\phi \simeq \epsilon_{\phi'} \simeq C_{\phi\phi'}/2$. In consequence, most points in Fig.1, lower right, are concentrated in the middle of the side corresponding to $O_\delta = 0$.

Finally, for values of the CHOOZ angle $\sim \sqrt{m_{\nu_2}^2 m_{\nu_1}/m_{\nu_3}^3}$, both terms in eq.(45) have to be taken into account. In this case, we cannot say that there is a single "leptogenesis phase": both $\delta - 2\phi$ and $\phi - \phi'$ are "leptogenesis phases". Concerning the contributions from the phases δ, ϕ and ϕ' to the CP asymmetry, it is apparent from eq.(45) that in a generic point the three contributions are going to be comparable. To be precise:

$$\begin{aligned} \epsilon_\delta &= \frac{1}{2} C_{\delta\phi'} \\ \epsilon_\phi &= \frac{1}{2} C_{\phi\phi'} \\ \epsilon_{\phi'} &= \frac{1}{2}(C_{\delta\phi'} + C_{\phi\phi'}) \end{aligned} \tag{48}$$

where,

$$\begin{aligned} C_{\delta\phi'} &\simeq -\frac{3y_1^2}{2\pi}\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta - \phi') \\ C_{\phi\phi'} &\simeq \frac{3y_1^2}{4\pi}\,\frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi') \quad . \end{aligned}$$

From these formulas, it is possible to understand the circular pattern that appears in Fig. 1, lower left, changing from triangular coordinates to Cartesian coordinates. We define the Cartesian axes setting the origin at the lower left vertex of the triangle, and we denote as x (y) the horizontal (vertical) axis. The change of variables read,

$$
\begin{aligned}
x &= \frac{2}{\sqrt{3}}\left[\frac{O_\delta^2}{2} + O_\phi^2\right] \\
y &= O_\delta^2 \quad ,
\end{aligned}
\tag{49}
$$

and using that $\epsilon_{\phi'} \simeq \epsilon_\delta + \epsilon_\phi$, we obtain, after some algebra, $(x - \frac{1}{\sqrt{3}})^2 + (y - \frac{1}{3})^2 \simeq \frac{1}{9}$, which is the equation of a circle centered in the barycentre of the triangle, with radius 1/3.

7 The general case

In the general case the number of unknown parameters involved is rather large (five phases, two angles in the V_L matrix and the CHOOZ angle), so the analysis is much more intricate since many different limits arise. However, we will see that only a few limits are distinct and physically interesting; the rest correspond to small regions in the parameter space that could arise in particular models, but that we will not consider, following the same bottom-up spirit as in the rest of the paper.

The different limits stem from the possible ways to expand the denominator in our approximate expression for the CP asymmetry

$$
\epsilon \simeq \frac{3y_1^2}{8\pi \sum_n |W_{1n}|^2 m_{\nu_n}^2} \mathrm{Im}\left\{\frac{\sum_i W_{1i}^2 m_{\nu_i}^3}{\sum_j W_{1j}^2 m_{\nu_j}}\right\}. \tag{50}
$$

The relevant elements in W for the calculation are

$$
\begin{aligned}
W_{11} &\simeq e^{-i\phi/2}\left[\frac{1}{\sqrt{2}}\, c_{12}^L c_{13}^L + e^{i\varphi_{12}} s_{12}^L c_{13}^L \left(\frac{1}{2} + \frac{1}{2} e^{i\delta} s_{13}\right) - e^{i\varphi_{13}} s_{13}^L \left(\frac{1}{2} - \frac{1}{2} e^{i\delta} s_{13}\right)\right] \\
W_{12} &\simeq e^{-i\phi'/2}\left[\frac{1}{\sqrt{2}}\, c_{12}^L c_{13}^L - e^{i\varphi_{12}} s_{12}^L c_{13}^L \left(\frac{1}{2} - \frac{1}{2} e^{i\delta} s_{13}\right) + e^{i\varphi_{13}} s_{13}^L \left(\frac{1}{2} + \frac{1}{2} e^{i\delta} s_{13}\right)\right] \\
W_{13} &\simeq e^{-i\delta} c_{12}^L c_{13}^L s_{13} - \frac{1}{\sqrt{2}}\, e^{i\varphi_{12}} s_{12}^L c_{13}^L - \frac{1}{\sqrt{2}}\, e^{i\varphi_{13}} s_{13}^L
\end{aligned}
\tag{51}
$$

where we have approximated $\cos\theta_{13} \simeq 1$ and we have assumed maximal solar and atmospheric mixings. It is apparent from these equations that different limits are going to arise depending on the mixing angles in V_L and the CHOOZ angle. We find then an interesting interplay between leptogenesis and lepton flavour violation, induced by radiative corrections through $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L$. However, from the parametrization we have chosen for V_L, eq.(21), one realizes that the off-diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ depend also on θ_{23}^L, that does not play any role in the CP asymmetry generated in the decay of the lightest right-handed neutrino. It is important, though, for the computation of the CP asymmetry generated in the decay of the next-to-lightest right-handed neutrino, that could be relevant, or even dominant, in some scenarios (particularly in scenarios with non-thermal creation of right handed neutrinos). Research along this lines would be certainly interesting, since in this case lepton flavour violation could be intimately related with leptogenesis.

We obtain simple analytic expressions when the denominator of eq. (50) can be expanded in small parameters. When $|W_{13}|^2 < |W_{12}|^2 m_{\nu_2}^2/m_{\nu_3}^2$ which will be the case if θ_{12}^L, θ_{13}^L, $\theta_{13} < .1$, ϵ can be approximated by

$$\epsilon \simeq \frac{3y_1^2}{8\pi|W_{12}|^2}\mathrm{Im}\left[\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3\frac{W_{13}^2}{W_{12}^2} - \left(\frac{m_{\nu_1}}{m_{\nu_2}}\right)\frac{W_{11}^2}{W_{12}^2}\right] \quad . \tag{52}$$

On the other hand, when the mixing in V_L is large, in the sense that $\sin\theta_{12}^L$ or $\sin\theta_{13}^L$ is larger than ~ 0.1, then $|W_{13}|^2 > |W_{12}|^2 m_{\nu_2}/m_{\nu_3}$, and the CP asymmetry reads

$$\epsilon \simeq -\frac{3y_1^2}{8\pi|W_{13}|^2}\left(\frac{m_{\nu_2}}{m_{\nu_3}}\right)\mathrm{Im}\left[\frac{W_{12}^2}{W_{13}^2}\right] \quad . \tag{53}$$

There is also an intermediate case, between these limits, where $|W_{13}|^2 m_{\nu_3}/m_{\nu_2} < |W_{12}|^2 < |W_{13}|^2 m_{\nu_3}^2/m_{\nu_2}^2$. We do not discuss this, because $m_{\nu_3}/m_{\nu_2} \sim 10$ for the hierarchical LMA solution we consider.

Let us analyze the two cases separately.

7.1 $|W_{11}| \sim |W_{12}| \gg |W_{13}| m_{\nu_3}/m_{\nu_2}$

The analysis for this case is parallel to the one we performed in the previous section for the case $V_L = 1$, where $|U_{11}| \sim |U_{12}| \gg |U_{13}|$. Using that $m_{\nu_2} \gg m_{\nu_1}$, $m_{\nu_3} s_i s_j$, where s_i is any of s_{13}, s_{12}^L, s_{13}^L, we can expand eq.(52), keeping the leading order terms in the expansion. The result is:

$$\begin{aligned}\epsilon \simeq -\frac{3y_1^2}{4\pi}\Bigg\{\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 \Big[&2s_{13}^2\sin(2\delta-\phi') - (s_{12}^L)^2\sin(2\varphi_{12}+\phi') - (s_{13}^L)^2\sin(2\varphi_{13}+\phi') \\ &-2\sqrt{2}s_{13}s_{12}^L\sin(\delta-\varphi_{12}-\phi') - 2\sqrt{2}s_{13}s_{13}^L\sin(\delta-\varphi_{13}-\phi') - 2s_{12}^L s_{13}^L\sin(\varphi_{12}+\varphi_{13}+\phi')\Big] \\ &-\left(\frac{m_{\nu_1}}{m_{\nu_2}}\right)\sin(\phi-\phi')\Bigg\} \quad .\end{aligned} \tag{54}$$

Obviously, in the limit $s_{12}^L, s_{13}^L \to 0$ we recover eq.(45). In the case $V_L = 1$ we found different limits, depending on the value of the CHOOZ angle. Now, the role of the CHOOZ angle is played by the angles θ_{12}^L, θ_{13}^L and the CHOOZ angle itself, and the results are different depending on the values of these angles compared with $\sqrt{m_{\nu_1} m_{\nu_2}^2/m_{\nu_3}^3}$.

- When *any* of the angles s_{13}, s_{12}^L or s_{13}^L is much larger than $\sqrt{m_{\nu_1} m_{\nu_2}^2/m_{\nu_3}^3}$, the term proportional to $(m_{\nu_3}/m_{\nu_2})^3$ in eq.(54) dominates:

$$\begin{aligned}\epsilon \simeq -\frac{3y_1^2}{4\pi}\left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 \Big[&2s_{13}^2\sin(2\delta-\phi') - (s_{12}^L)^2\sin(2\varphi_{12}+\phi') - (s_{13}^L)^2\sin(2\varphi_{13}+\phi') \\ &-2\sqrt{2}s_{13}s_{12}^L\sin(\delta-\varphi_{12}-\phi') - 2\sqrt{2}s_{13}s_{13}^L\sin(\delta-\varphi_{13}-\phi') - 2s_{12}^L s_{13}^L\sin(\varphi_{12}+\varphi_{13}+\phi')\Big] \quad .\end{aligned} \tag{55}$$

We recall here that this limit corresponds to the case where the lightest neutrino mass is very small. On the other hand, for the mass hierarchy that we are using as reference to present our numerical results, $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$, this limit arises when any of the angles is much larger than ~ 0.01, in particular, when the CHOOZ angle is close to the experimental bound and the relevant angles in V_L are comparable to or smaller than the CHOOZ angle.

When the three angles are comparable in size, we see that there are three "leptogenesis phases": $2\delta - \phi'$, $2\varphi_{12} + \phi'$ and $2\varphi_{13} + \phi'$ (the arguments of the sines in the last three terms of eq.(55) are combinations of these). Notice that in this limit ϕ' is an important phase for leptogenesis, although it cannot be regarded as the "leptogenesis phase", since the actual "leptogenesis phases" are combinations of ϕ' with other phases. However, an indication for a non-vanishing ϕ', coming for example from experiments on neutrino-less double beta decay, would provide an indication for leptogenesis.

If there are two angles that are comparable, while the third is much smaller than the others, then there are two "leptogenesis phases". To understand better the results for this limit, we analyze in some detail the case $s_{13} \simeq s^L_{12} \gg s^L_{13}$. If s^L_{23} is also small, this case would produce small rates for $\mu \to e\gamma$, as can be checked from eq.(21). The results for the other possibilities, $s_{13} \simeq s^L_{13} \gg s^L_{12}$ and $s^L_{12} \simeq s^L_{13} \gg s_{13}$, can be easily deduced from this analysis, making the appropriate substitutions. We have computed numerically the different contributions to the CP asymmetry for the choice of angles $s_{13} = s^L_{12} = 0.03$, $s^L_{13} = 0$, the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$ and assigning random values, between 0 and 2π, to the phases. We obtain that for most of the parameter space, the only non-vanishing contributions to the CP asymmetry are ϵ_δ, $\epsilon_{\varphi_{12}}$ and $\epsilon_{\phi'}$ (φ_{13} does not play any role, because we have set s^L_{13} to 0). Since there are essentially only three contributions involved, a convenient way of presenting the results is using a triangular plot. In Fig.2, left, we explain how to interpret the distances to the different sides of the triangle, whereas in Fig.2, right, we show the numerical results of the calculation. We find that in general the three contributions are comparable, although the contribution from ϕ' is slightly larger than the other two. This can be understood from the analytical approximation, eq.(55), setting $s^L_{13} = 0$. The different contributions to the CP asymmetry are:

$$\begin{aligned} \epsilon_\delta &= \frac{1}{2} C_{\delta\phi'} + \frac{1}{3} C_{\delta\phi'\varphi_{12}} \\ \epsilon_{\phi'} &= \frac{1}{2}(C_{\delta\phi'} + C_{\phi'\varphi_{12}}) + \frac{1}{3} C_{\delta\phi'\varphi_{12}} \\ \epsilon_{\varphi_{12}} &= \frac{1}{2} C_{\phi'\varphi_{12}} + \frac{1}{3} C_{\delta\phi'\varphi_{12}} \quad , \end{aligned} \tag{56}$$

where,

$$\begin{aligned} C_{\delta\phi'} &\simeq -\frac{3y_1^2}{2\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta - \phi') \\ C_{\phi'\varphi_{12}} &\simeq \frac{3y_1^2}{4\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 (s^L_{12})^2 \sin(2\varphi_{12} + \phi') \end{aligned} \tag{57}$$

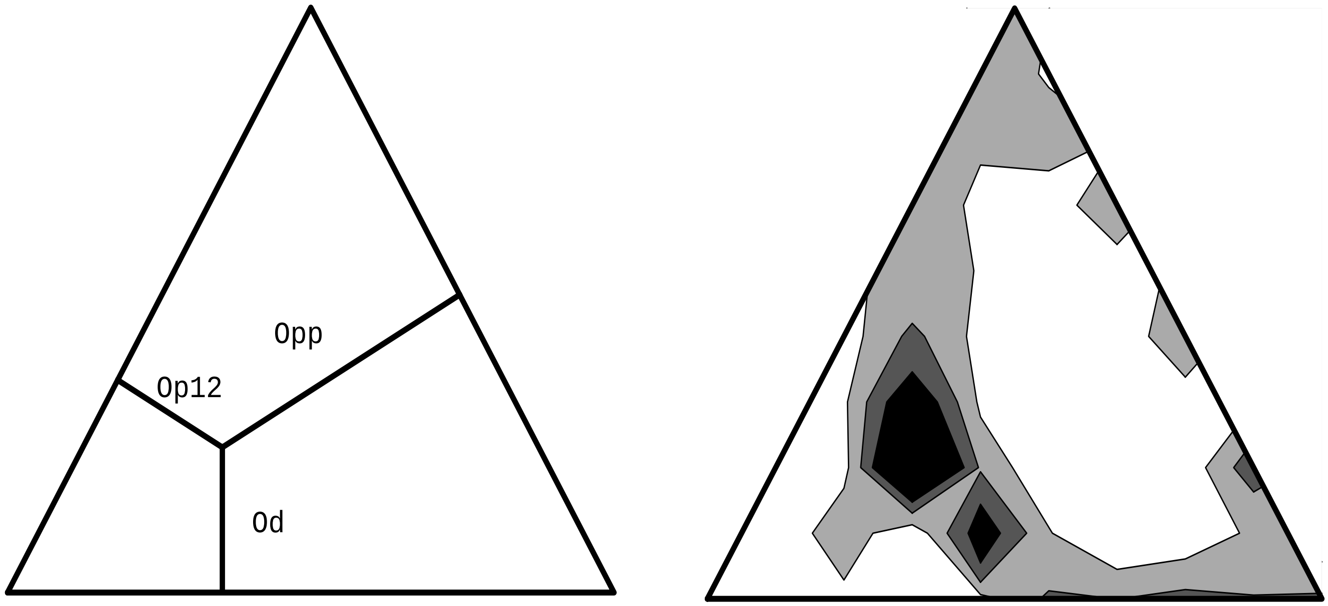

Figure 2: The same as Fig.1, for the case where $s_{13} = s_{12}^L = 0.03$, $s_{13}^L = 0$. The left plot indicates how to interpret the distances to the different sides of the triangle, and the right plot shows a density plot of the "phase overlaps" for random values of the phases. The darkest regions correspond to the largest density of points.

$$C_{\delta\phi'\varphi_{12}} \simeq -\frac{3y_1^2}{\sqrt{2}\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13} s_{12}^L \sin(\delta - \varphi_{12} - \phi'),$$

that are in general comparable.

Finally, if one of the angles dominates over the others, the conclusions are very similar as for the case $V_L = 1$, where the CP asymmetry received contributions from ϕ' and δ. Here, the role of δ is played by the phase corresponding to the angle that dominates (δ for s_{13}, φ_{12} for s_{12}^L, and φ_{13} for s_{13}^L). In this case, $\epsilon \simeq C_{x\phi'}$, where x is the relevant angle among δ, φ_{12} and φ_{13}. On the other hand, the normalized contributions are $O_x \simeq O_{\phi'} \simeq 1/\sqrt{2}$, while they are vanishing for the rest of the phases. For example, if $s_{12}^L \gg s_{13}, s_{13}^L$, then $\epsilon \simeq C_{\phi'\varphi_{12}}$ and $O_{\varphi_{12}} \simeq O_{\phi'} \simeq 1/\sqrt{2}$.

• When *all* the angles (s_{13}, s_{12}^L and s_{13}^L) are much smaller than $\sqrt{m_{\nu_1} m_{\nu_2}^2 / m_{\nu_3}^3}$, the term proportional to m_{ν_1}/m_{ν_2} dominates in eq.(54) and the CP asymmetry reads

$$\epsilon \simeq \frac{3y_1^2}{4\pi} \left(\frac{m_{\nu_1}}{m_{\nu_2}}\right) \sin(\phi - \phi') \quad . \tag{58}$$

In this limit, the results are identical as in the corresponding limit in the case $V_L = 1$, and there is a single "leptogenesis phase", $\phi - \phi'$. So, the normalized contributions to the CP asymmetry from ϕ and ϕ' are equal to $1/\sqrt{2}$, while the contributions from the rest of the phases vanish. The numerical analysis yield a plot that is very similar to Fig.1, lower right, where the role of O_δ is played by either $O_{\varphi_{12}}$, $O_{\varphi_{13}}$ or O_δ.

• Lastly, in the situations where the two terms in eq.(54) are comparable, the analysis is very involved, since in principle there are four independent "leptogenesis phases", namely, $2\delta - \phi'$, $2\varphi_{12} - \phi'$, $2\varphi_{13} - \phi'$ and $\phi - \phi'$. So, the CP asymmetry receives contributions from the five phases, and in general they are comparable in size. Hence, it is very difficult to extract any general conclusion for this case.

7.2 $|W_{12}|^2 m_{\nu_2}/m_{\nu_3} < |W_{13}|^2$

For simplicity, and since the number of phases and angles involved is rather large, we will set one of the angles in V_L equal to zero, say $s^L_{13} = 0$, so the phase φ_{13} becomes irrelevant. Since s^L_{12} and s^L_{13} appear in a similar way in the formulas, one can qualitatively derive the result when s^L_{13} is different from zero. With this choice, we are left with only two angles, the CHOOZ angle, s_{13}, and one angle in V_L, s^L_{12}. The limit we are studying in this section requires s^L_{12} larger than ~ 0.1. Then, using the experimental bound on the CHOOZ angle and that our phases are generically of order 1, the denominator can be expanded as

$$\frac{1}{|W_{13}|^2} \simeq \frac{2}{(s^L_{12})^2}\left(1 + 2\sqrt{2}\frac{c^L_{12}}{s^L_{12}} s_{13}\cos(\delta + \varphi_{12})\right) \quad . \tag{59}$$

Hence, the CP asymmetry can be approximated by

$$\begin{aligned} \epsilon \simeq -\frac{3y_1^2}{4\pi}\frac{(c^L_{12})^3}{(s^L_{12})^5}\Bigg\{ &\left(\frac{s^L_{12}}{c^L_{12}}\right)\left[-2\sin(2\varphi_{12}+\phi') + 2\sqrt{2}\left(\frac{s^L_{12}}{c^L_{12}}\right)\sin(\varphi_{12}+\phi') - \left(\frac{s^L_{12}}{c^L_{12}}\right)^2 \sin\phi'\right] \\ &+ s_{13}\Bigg[2\sqrt{2}[\sin(\delta-\varphi_{12}-\phi') - 3\sin(\delta+3\varphi_{12}+\phi')] \\ &-4\left(\frac{s^L_{12}}{c^L_{12}}\right)[\sin(\delta-\phi') - 3\sin(\delta+2\varphi_{12}+\phi')] + \sqrt{2}\left(\frac{s^L_{12}}{c^L_{12}}\right)^2 [2\sin(\delta-\varphi_{12}-\phi') \\ &+ \sin(\delta+\varphi_{12}-\phi') - 3\sin(\delta+\varphi_{12}+\phi')] - 2\left(\frac{s^L_{12}}{c^L_{12}}\right)^3 \sin(\delta-\phi')\Bigg]\Bigg\} \quad . \end{aligned} \tag{60}$$

This expression is rather cumbersome and it is difficult to extract information from it. It is not possible in general to identify the "leptogenesis phase", although it is clear that leptogenesis depends mainly on ϕ' and φ_{12}, whereas the dependence on δ is weaker.

In Fig.3 we show the numerical results for this case. As usual, we show a triangle with the meaning of the distances to the different sides (upper left plot), and density plots of the "phase overlaps" for the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$, and taking random values for the phases between 0 and 2π. In the upper right (lower left) plot we show the results for $\tan\theta^L_{12} = 0.5$ (1) and $s_{13} = 0.1$. In both plots, the points are concentrated close to the base of the triangle (that corresponds to O_δ small), due to the small value of the CHOOZ angle. In the plot corresponding to $\tan\theta^L_{12} = 0.5$

the points are concentrated around the center of the base, whereas for $\tan\theta^L_{12} = 1$, they are spread all over the base. This can be understood from the dependence of ϵ on $\cot\theta^L_{12}$. For $\tan\theta^L_{12} = 0.5$, the terms with both φ_{12} and ϕ' are the dominant ones, so $C_{\phi'\varphi_{12}}$ is the largest contribution to the CP asymmetry. On the other hand, when $\tan\theta^L_{12} = 1$ these terms are comparable to the one with $\sin\phi'$, so ϵ is dominated by $C_{\phi'\varphi_{12}}$ and $C_{\phi'}$. Depending on the value of ϕ' the points spread along the basis of the triangle. In the lower right plot we show the numerical results for $\tan\theta^L_{12} = 0.5$ and $s_{13} = 0.01$. The plot is similar to the one with $s_{13} = 0.1$ but with an even smaller value of O_δ.

8 Summary and Discussion

If neutrino masses are due to the seesaw mechanism, then the heavy right-handed neutrinos can generate a lepton asymmetry in the early Universe when they decay out of equilibrium, if they have CP violating couplings. Such complex couplings in the high energy parameters could induce three phases in the light neutrino sector, called ϕ, ϕ' and δ (these are the phases that appear in the MNS matrix; see eqs. (7) and (8)). Upcoming experiments may be sensitive to two of these phases: the Dirac phase δ could be measured at a neutrino factory, whereas the Majorana phase ϕ' might have some observable effects in neutrino-less double beta decay. In this paper, we are interested in the relative importance of the phases ϕ' and δ for leptogenesis.

To address this issue, we use a parametrization of the seesaw in terms of weak scale variables: the light neutrino masses, the MNS matrix, the eigenvalues of the neutrino Yukawa matrix, and a unitary matrix V_L. We assume a hierarchical light neutrino spectrum, with the lightest neutrino mass of order $m_{\nu_3}/100$, and an MNS matrix that corresponds to the LAMSW solution to the solar neutrino problem. The matrix V_L is related to the off-diagonal (lepton flavour violating) elements of the slepton mass matrix, and contains three phases, two of which $(\varphi_{12}, \varphi_{13})$ are relevant for our calculation. It is important to use a parametrization in terms of "physical" weak scale phases; this is discussed in section 5.

In the parameter space we are interested in, we find a simple analytic approximation for the lepton asymmetry ϵ_1, produced in the decay of the *lightest* right-handed neutrino ν_{R_1}:

$$\epsilon \simeq \frac{3y_1^2}{8\pi\sum_j |W_{1j}|^2 m^2_{\nu_j}} \mathrm{Im}\left\{\frac{\sum_k W^2_{1k} m^3_{\nu_k}}{\sum_n W^2_{1n} m_{\nu_n}}\right\} \tag{61}$$

(see eq. (30)). In this equation, y_1 is the smallest eigenvalue of the neutrino Yukawa matrix, and W is the unitary transformation from the basis where the ν_L mass matrix is diagonal to the basis where $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is diagonal: $W_{1n} = [V_L]_{1m}[U]_{mn}$ where V_L and U are defined in eqs. (7), (8) and (21).

We are interested in the *relative* importance of the phases ϕ' and δ for ϵ. That is, we do not discuss whether we get ϵ large enough, which is essentially controlled by real parameters, such as y_1. We assume that the observed baryon asymmetry is produced in the out of equilibrium decay of ν_{R_1} and study how important could be δ or ϕ' for

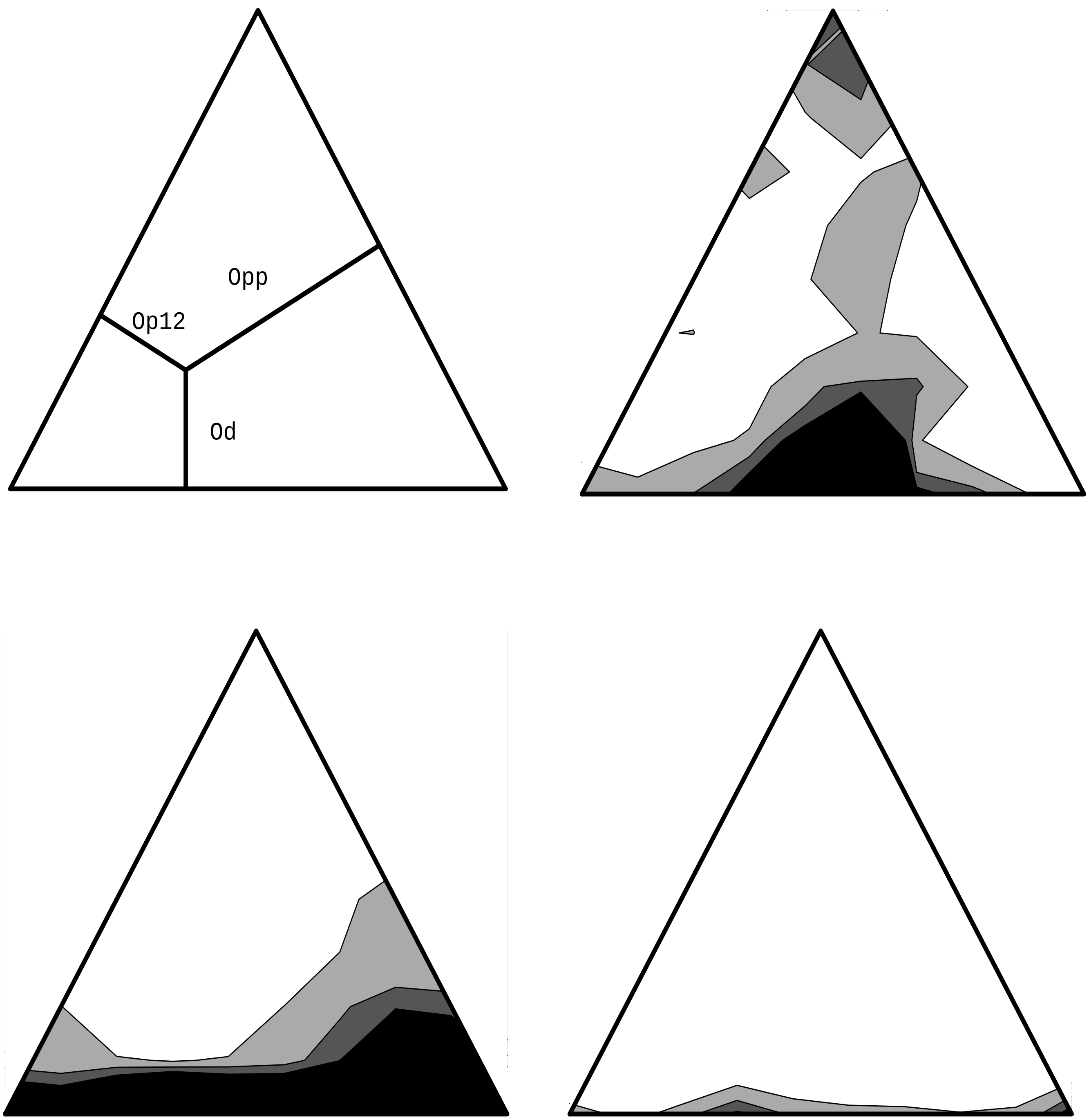

Figure 3: The same as fig.1 for different situations where $|W_{12}|^2 m_{\nu_2}/m_{\nu_3} < |W_{13}|^2$. The upper left plot indicates the meaning of the distances to the different sides of the triangle. The rest of the triangles show density plots of the "phase overlaps" for random values of the phases, $s^L_{13} = 0$ and $\tan\theta^L_{12} = 0.5$, $s_{13} = 0.1$ (upper right), $\tan\theta^L_{12} = 1$, $s_{13} = 0.1$ (lower left), and $\tan\theta^L_{12} = 0.5$, $s_{13} = 0.01$ (lower right). The darkest regions correspond to the largest density of points. (See sect. 7.2 for details.)

the CP asymmetry. In other words, if we suppose that ϵ is of the correct size, what fraction of ϵ is due to δ or ϕ'?

We Fourier expand ϵ on the five relevant phases of our low-energy parametrization

$$\epsilon = \sum_{j,k,l,m,n} A_{jklmn} \sin(j\delta + k\phi + l\phi' + m\varphi_{12} + n\varphi_{13}) \quad , \tag{62}$$

and then divide the sum into five components, one due to each phase. In ϵ_δ, which is the component due to δ, we put all the terms from the Fourier expansion that are $\propto \sin(j\delta)$. We define $C_{\delta\alpha}$ to be the sum of all the terms $\propto \sin(j\delta + n\alpha)$, and divide it equally between ϵ_δ and ϵ_α — that is, we add $\frac{1}{2}\sum_\alpha C_{\delta\alpha}$ to ϵ_δ. We also add the terms $\propto \sin(j\delta+n\alpha+m\beta)$, multiplied by 1/3, and so on (α and β are one of $\{\phi, \phi', \varphi_{12}, \varphi_{13}\}$). This procedure is described in more detail in section 5, and the formula for ϵ_δ can be found in eq.(37). Then we define a normalized "fraction of ϵ due to δ", which we call the overlap between the leptogenesis phase and δ, as

$$O_\delta = \frac{|\epsilon_\delta|}{\sqrt{\sum_\alpha \epsilon_\alpha^2}} \quad . \tag{63}$$

The magnitude of O_δ or $O_{\phi'}$ depends on five phases and three unknown real parameters: the CHOOZ angle, θ_{13}, and two angles from V_L, that is related to radiative decays. In the numerical calculation, we fix the real angles and assign random values (linearly distributed) to the phases between 0 and 2π. The numerical results are shown in density plots in the $O_\delta - O_{\phi'}$ space.

We present results for two representative cases. In section 6 we discuss the $V_L = 1$ case, where the three relevant phases are δ, ϕ' and ϕ (the phases of the MNS matrix), and in Section 7 we allow for two non-zero angles in V_L. For the sake of clarity in the presentation, in Section 7 we analyze simplified scenarios to reduce the number of phases involved to three. Since the definition of overlap satisfies the identity $\sum O_\alpha^2 = 1$, a convenient way of showing the results is by using a triangular plot, where the distance to each side of the triangle corresponds to O_α^2.

The $V_L = 1$ model should be a good approximation when the angles in V_L are smaller than the CHOOZ angle. It can be checked from eqs.(51) and (61) that the Dirac phases δ, φ_{12} and φ_{13} appear in ϵ multiplied by the sine of a real angle (θ_{13}, θ^L_{12} and θ^L_{13}, respectively). If θ^L_{12} and θ^L_{13} are much smaller than the CHOOZ angle θ_{13}, then φ_{12} and φ_{13} are less important for ϵ than δ.

In Section 7 we analyze the situation where there are angles in V_L larger than the CHOOZ angle. The angles of V_L are related to the branching ratios for $\ell_j \to \ell_i\gamma$, as discussed after eq. (22) : $BR(\ell_j \to \ell_i\gamma) \propto |[V_L]^*_{3j}[V_L]_{3i}|^2$ Current limits on $\tau \to \mu\gamma$, $\tau \to e\gamma$, and $\mu \to e\gamma$ are satisfied if all angles $\theta^L_{ij} \lesssim .1$. However, present bounds and anticipated improvements on all three branching ratios can be satisfied if *e.g.* $\theta^L_{13}, \theta^L_{23} \simeq 0$ and $\theta^L_{12} \sim 1$. So it is phenomenologically possible to have at least one large angle. The associated phase could then be important for leptogenesis; this possibility is studied in that section.

The approximate expression for ϵ, eq.(61), shows that for generic W_{1j}, $\epsilon \propto \mathrm{Im}(W_{12}W^*_{13})^2$. That is, the terms proportional to m_{ν_1} can be neglected when $W_{12} \sim W_{13}$, as discussed in section 7. Recall that W_{13} contains terms $\sim \sin\theta_{13}$, $\sin\theta^L_{12}$, $\sin\theta^L_{13}$. We set $\sin\theta^L_{13}$ to zero, to ensure that $\ell_j \to \ell_i\gamma$ constraints are satisfied, and because the functional dependence of ϵ on $\sin\theta^L_{12}$ and $\sin\theta^L_{13}$ is similar [4]. So the case studied in that section has three phases: δ, ϕ' and φ_{12}.

We find that the Majorana phase ϕ' is (almost) always important for leptogenesis. For instance, the fraction of ϵ that is due to ϕ', $O_{\phi'}$, is significant in all the cases we have studied. Algebraically, the reason is that the main contribution to ϵ in eq.(61) is generically proportional to W_{12}, that is proportional to $e^{i\phi'}$, unless some cancellations occur. The contribution from any of the other phases can be suppressed by sending a small parameter to zero. The Majorana phase of m_{ν_1}, ϕ, becomes unimportant as $m_{\nu_1} \to 0$, and the three Dirac phases δ, φ_{12} and φ_{13} multiply angles which are positively small (s_{13}) or believed to be small (s^L_{12}, s^L_{13}). The contribution of ϕ', on the other hand, is consistently significant.

It is interesting to study how closely related are the Majorana phases of the light neutrinos to the Majorana phases of the heavy right-handed neutrinos. It is well known that when neutrinos have Majorana masses, there is CP violation even in the two generation model. This suggests that the Majorana phases of the ν_R sector could be more important for leptogenesis than the Dirac phase, because they can contribute to the CP asymmetry ϵ suppressed by mixing between only two of the ν_R, rather than mixing among the three ν_R, as required for the Dirac phase. However, there is no symmetry-based distinction between Majorana phases and Dirac phases. The high scale Majorana phases are functions of all the weak scale parameters—the real ones as well as all the Dirac and Majorana phases. So the reason ϕ' is important is not that low energy Majorana phases determine the high scale Majorana phases. On can check from the formulae in section 4 that ϕ' is usually significant because it multiples a not-very-small mass, rather than a (possibly) small mixing angle.

We find that the phase δ, the phase that neutrino factories could measure, *can* be important for leptogenesis. When the CHOOZ angle is large enough to detect CP violation at neutrino factories (which requires $\theta_{13} \gtrsim .01$), δ contributes significantly to the leptogenesis phase ($|O_\delta|^2 \gtrsim .3$) provided that the off-diagonal elements of the slepton mass matrix are small, or more precisely, that $\theta^L_{12},\ \theta^L_{13} \lesssim \theta_{13}$. This can be seen from our low-energy approximation to ϵ, eq. (61): if W_{13} is not too small, ϵ depends on the phase difference between W_{12} and W_{13}, and if $\theta_{13} \gtrsim \theta^L_{12},\ \theta^L_{13}$, the phase of W_{13} is δ. Although δ appears always suppressed by the CHOOZ angle, which is small, it plays an important role, because W_{13} is multiplied by the largest neutrino mass m_{ν_3}. So it is significant, unless $\theta^L_{12},\ \theta^L_{13} \gtrsim \theta_{13}$ (see fig 3) The branching ratios $\ell_j \to \ell_i\gamma$ depend on the θ^L_{ij}, as discussed in section 4. Measuring these branching ratios could determine the size of some of the θ^L_{ij} relative to the CHOOZ angle θ_{13}, and therefore the probable importance of δ. However, lepton flavour conservation in the charged lepton sector, does *not* imply that O_δ is large. Even if none of the decays $\ell_j \to \ell_i\gamma$ are observed, θ^L_{12} can still be large, making φ_{12} more relevant for leptogenesis than δ.

We do not find a simple correlation between the sign of low energy phases and the sign of the CP asymmetry ϵ. Such a correlation would be interesting, and exists in

[4]this is because $\nu_3 \simeq (\nu_\tau + \nu_\mu)/\sqrt{2}$

certain models [41]. However, in our bottom-up approach, ϵ is usually proportional to phase differences ($\epsilon \sim \sin(j\alpha + n\beta)$), as can be seen from eq. (61) and the various limiting cases discussed in sections 6 and 7.

In summary, we have studied the relative importance of low energy phases for leptogenesis. Using a parametrization of the seesaw mechanism in terms of weak scale variables, we express the CP asymmetry produced in the decay of the lightest ν_R as a function of the "neutrino factory phase" δ, the "neutrinoless double beta decay phase" ϕ', and three other "physical" weak scale phases. We introduce a way of splitting ϵ into contributions due to the different phases, δ, ϕ', etc. We assume that ϵ is big enough to be responsible for the observed baryon asymmetry, and compare the relative size of the different contributions. We find that ϕ' is generically important for leptogenesis. The importance of δ depends on the mixing angles of the slepton sector. If these are smaller than the CHOOZ angle, then δ makes a significant contribution to leptogenesis.

Acknowledgments

We thank Michael Plümacher and Graham Ross for discussions, and Wilfred Buchmüller, F Schrempp and particularily Laura Covi for comments and suggestions.

Addendum

After this work was completed, a related analysis appeared [46].

Appendix

In the appendix we explain the numerical procedure that we have followed to compute the contributions to the CP asymmetry from the different phases. For the sake of clarity we will only present the procedure we followed for the case $V_L = 1$, where only the phases δ, ϕ and ϕ' were relevant. The extension to the general case is straight-forward.

Our starting point to compute the contributions was the Fourier expansion of the CP asymmetry

$$\epsilon = \sum_{j,k,l} A_{jkl} \sin(j\delta + k\phi + l\phi') = C_\delta + C_\phi + C_{\phi'} + C_{\delta\phi} + C_{\delta\phi'} + C_{\phi\phi'} + C_{\delta\phi\phi'} \quad (64)$$

with C_α, $C_{\alpha\beta}$ and $C_{\delta\phi\phi'}$ as in eqs.(34)-(36). From the periodicity of ϵ it is apparent that

$$\begin{aligned} C_\delta &= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\phi \; d\phi' \; \epsilon \\ C_\phi &= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\delta \; d\phi' \; \epsilon \\ C_{\phi'} &= \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\delta \; d\phi \; \epsilon \end{aligned} \quad (65)$$

$$
\begin{aligned}
C_{\delta\phi} &= \frac{1}{(2\pi)}\int_{-\pi}^{\pi} d\phi'\ \epsilon - (C_\delta + C_{\phi'}) \\
C_{\delta\phi'} &= \frac{1}{(2\pi)}\int_{-\pi}^{\pi} d\phi\ \epsilon - (C_\delta + C_{\phi}) \\
C_{\phi\phi'} &= \frac{1}{(2\pi)}\int_{-\pi}^{\pi} d\delta\ \epsilon - (C_\phi + C_{\phi'})
\end{aligned}
\tag{66}
$$

$$
C_{\delta\phi\phi'} = \epsilon - (C_\delta + C_\phi + C_{\phi'} + C_{\delta\phi} + C_{\delta\phi'} + C_{\phi\phi'}) \quad . \tag{67}
$$

These integrals can be computed numerically, thus giving the different contributions to the CP asymmetry. This avoids difficulties with the points where the approximation eq. (30) breaks down. However, it is also interesting to solve this integrals analytically, to cross-check the results we obtained in Section 6. The results for the double integrals are

$$
C_\delta \simeq 0 \qquad C_\phi \simeq 0 \qquad C_{\phi'} \simeq 0. \tag{68}
$$

On the other hand, the results for the single integrals is more involved and depends on the particular point of the parameter space. These integrals can be computed using the residue theorem; the number of poles inside the unit circle depends on the values of the phases and other neutrino parameters, especially on the CHOOZ angle, hence the dependence of the result on the chosen parameters. However, some care must be exercised in using the residue theorem, because there can be poles in eq. (30) at points where the approximation breaks down. Such poles must be neglected.

The results for the single integrals are different depending on the CHOOZ angle. We consider three possibilities:

• When the CHOOZ angle is close to the experimental upper limit, or to be precise, when

$$
\begin{aligned}
|m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &< m_{\nu_2}\ c_{13}^2 s_{12}^2 \\
|m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &> m_{\nu_1}\ c_{13}^2 c_{12}^2 \\
|m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'}| &< m_{\nu_3}\ s_{13}^2 \quad ,
\end{aligned}
\tag{69}
$$

the single integrals read

$$
\begin{aligned}
& C_{\delta\phi} \simeq 0 \\
& C_{\delta\phi'} \simeq -\frac{3y_1^2}{8\pi D}\ \mathrm{Im}\left\{\frac{m_{\nu_2}^3\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}^3\ s_{13}^2\ e^{2i\delta}}{m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}}\right\} \simeq -\frac{3y_1^2}{2\pi}\ \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta - \phi') \\
& C_{\phi\phi'} \simeq 0 \quad ,
\end{aligned}
\tag{70}
$$

where D was defined after eq.(44). This result, coincide with eq.(46), that was obtained using a completely different method.

• When the CHOOZ angle is very small, or when the conditions

$$
\begin{aligned}
|m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &< m_{\nu_2}\ c_{13}^2 s_{12}^2 \\
|m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &< m_{\nu_1}\ c_{13}^2 c_{12}^2 \\
|m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'}| &> m_{\nu_3}\ s_{13}^2
\end{aligned}
\tag{71}
$$

are fulfilled, the results for the single integrals are

$$\begin{aligned} C_{\delta\phi} &\simeq 0 \\ C_{\delta\phi'} &\simeq 0 \\ C_{\phi\phi'} &\simeq -\frac{3y_1^2}{8\pi D}\,\mathrm{Im}\left\{\frac{m_{\nu_1}^3\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}^3\ c_{13}^2 s_{12}^2\ e^{i\phi'}}{m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'}}\right\} \simeq \frac{3y_1^2}{4\pi}\ \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi-\phi') \quad . \end{aligned} \tag{72}$$

This result is identical to the result obtained using series expansions in Section 6, eq.(47).

• For intermediate values of the CHOOZ angle, it is usually the case that

$$\begin{aligned} |m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &< m_{\nu_2}\ c_{13}^2 s_{12}^2 \\ |m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}| &> m_{\nu_1}\ c_{13}^2 c_{12}^2 \\ |m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi}| &> m_{\nu_3}\ s_{13}^2 \quad , \end{aligned} \tag{73}$$

so the single integrals are

$$\begin{aligned} C_{\delta\phi} &\simeq 0 \\ C_{\delta\phi'} &\simeq -\frac{3y_1^2}{8\pi D}\,\mathrm{Im}\left\{\frac{m_{\nu_2}^3\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}^3\ s_{13}^2\ e^{2i\delta}}{m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'} + m_{\nu_3}\ s_{13}^2\ e^{2i\delta}}\right\} \simeq -\frac{3y_1^2}{2\pi}\ \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta-\phi') \\ C_{\phi\phi'} &\simeq -\frac{3y_1^2}{8\pi D}\,\mathrm{Im}\left\{\frac{m_{\nu_1}^3\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}^3\ c_{13}^2 s_{12}^2\ e^{i\phi'}}{m_{\nu_1}\ c_{13}^2 c_{12}^2\ e^{i\phi} + m_{\nu_2}\ c_{13}^2 s_{12}^2\ e^{i\phi'}}\right\} \simeq \frac{3y_1^2}{4\pi}\ \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi-\phi') \quad , \end{aligned} \tag{74}$$

that are identical to eq.(49).

References

[1] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].

[2] *see e.g.* W. Buchmuller and S. Fredenhagen, hep-ph/0001098; A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49** (1999) 35; V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk **166** (1996) 493 [Phys. Usp. **39** (1996) 461].

[3] R. J. Davis, D. S. Harmer and K. C. Hoffman, Phys. Rev. Lett. **20** (1968) 1205.

[4] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1562, Phys. Rev. Lett. **82** (1999) 1810, Phys. Rev. Lett. **82** (1999) 2430.

[5] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[6] M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. Van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedinds of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, ed.s A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys.Rev.Lett.* **44** (1980)912, *ibid. Phys.Rev.* **D23** (1981) 165.

[7] G. C. Branco, T. Morozumi, B. M. Nobre and M. N. Rebelo, arXiv:hep-ph/0107164.

[8] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, arXiv:hep-ph/0202030.

[9] J. R. Ellis, J. Hisano, S. Lola and M. Raidal, arXiv:hep-ph/0109125.

[10] S. Davidson and A. Ibarra, JHEP **0109** (2001) 013 [arXiv:hep-ph/0104076].

[11] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301;

[12] Q. R. Ahmad *et al.* [SNO Collaboration], arXiv:nucl-ex/0204008. Q. R. Ahmad *et al.* [SNO Collaboration], arXiv:nucl-ex/0204009.

[13] V. Barger, D. Marfatia, K. Whisnant and B. P. Wood, Phys. Lett. B **537** (2002) 179 [arXiv:hep-ph/0204253]. A. Bandyopadhyay, S. Choubey, S. Goswami and D. P. Roy, arXiv:hep-ph/0204286. J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, arXiv:hep-ph/0204314. P. C. de Holanda and A. Y. Smirnov, arXiv:hep-ph/0205241. A. Strumia, C. Cattadori, N. Ferrari and F. Vissani, arXiv:hep-ph/0205261. G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, arXiv:hep-ph/0206162.

[14] B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505; Y. Fukuda *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **77** (1996) 1683; S. Hatakeyama *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 2016; W. W. Allison *et al.*, Phys. Lett. B **391** (1997) 491; W. W. Allison *et al.* [Soudan-2 Collaboration], Phys. Lett. B **449** (1999) 137; W. Hampel *et al.* [GALLEX Collaboration], Phys. Lett. B **388** (1996) 384; D. N. Abdurashitov *et al.*, Phys. Rev. Lett. **77** (1996) 4708.

[15] M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **466** (1999) 415.

[16] see *e.g.* A.Alessandrello et. al, *Phys. Lett.* **B486** (2000) 13; Heidelberg-Moscow Collaboration, *Phys. Rev. Lett.* 83 (1999) 41-44; Heidelberg-Moscow Collaboration, *Nucl. Phys.* **A**694 (2001) 269-294.

[17] V. M. Lobashev *et al.*, Phys. Lett. B **460** (1999) 227. C. Weinheimer *et al.*, Phys. Lett. B **460** (1999) 219.

[18] *see, e.g.* C. Albright *et al.,* hep-ex/0008064
or, on the web: http://www.cap.bnl.gov/mumu/mu_home_page.html, or http://muonstoragerings.web.cern.ch/muonstoragerings/

[19] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. B **579** (2000) 17 [Erratum-ibid. B **593** (2001) 731] [arXiv:hep-ph/0002108].

[20] A. Romanino, Nucl. Phys. B **574** (2000) 675 [arXiv:hep-ph/9909425]. J. Burguet-Castell, M. B. Gavela, J. J. Gomez-Cadenas, P. Hernandez and O. Mena, Nucl. Phys. B **608** (2001) 301 [arXiv:hep-ph/0103258].

[21] V. Barger, S. L. Glashow, P. Langacker and D. Marfatia, arXiv:hep-ph/0205290.

[22] *see e.g.* S. R. Elliott and P. Vogel, arXiv:hep-ph/0202264.

[23] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961.

[24] J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 [arXiv:hep-ph/0103065].

[25] *see e.g.*W. Buchmuller and M. Plumacher, Phys. Rept. **320** (1999) 329, *and references therein.*

[26] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[27] J. Ellis, M. E. Gomez, G. K. Leontaris, S. Lola and D. V. Nanopoulos, Eur. Phys. J. C **14** (2000) 319; M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados, Phys. Rev. D **59** (1999) 116009; J. L. Feng, Y. Nir and Y. Shadmi, Phys. Rev. D **61** (2000) 113005. G. K. Leontaris and N. D. Tracas, Phys. Lett. B **431** (1998) 90; W. Buchmuller, D. Delepine and L. T. Handoko, Nucl. Phys. B **576** (2000) 445. W. Buchmuller, D. Delepine and F. Vissani, Phys. Lett. B **459** (1999) 171; D. F. Carvalho, M. E. Gomez and S. Khalil, hep-ph/0101250; J. Sato and K. Tobe, hep-ph/0012333; S. F. King and M. Oliveira, Phys. Rev. D **60** (1999) 035003; R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B **445** (1995) 219.

[28] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53** (1996) 2442. J. Hisano and D. Nomura, Phys. Rev. D **59** (1999) 116005. J. Sato, K. Tobe and T. Yanagida, Phys. Lett. B **498** (2001) 189. J. Hisano, D. Nomura, Y. Okada, Y. Shimizu and M. Tanaka, Phys. Rev. D **58** (1998) 116010; J. Hisano, D. Nomura and T. Yanagida, Phys. Lett. B **437** (1998) 351; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Lett. B **391** (1997) 341; P. Ciafaloni, A. Romanino and A. Strumia, Nucl. Phys. B **458** (1996) 3; G. Barenboim, K. Huitu and M. Raidal, Phys. Rev. D **63** (2001) 055006;

[29] S. Lavignac, I. Masina and C. A. Savoy, arXiv:hep-ph/0106245.

[30] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83** (1999) 1521 [arXiv:hep-ex/9905013].

[31] L.M. Barkov *et al.,* Research proposal for an experiment at PSI: R-99-05.1 . L. Serin, R Stroynowski, ATLAS internal note.

[32] J. Sato, K. Tobe and T. Yanagida, Phys. Lett. B **498** (2001) 189 [arXiv:hep-ph/0010348].

[33] G. C. Branco, L. Lavoura and M. N. Rebelo, Phys. Lett. B **180** (1986) 264. A. Santamaria, Phys. Lett. B **305** (1993) 90 [arXiv:hep-ph/9302301].

[34] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[35] S. Davidson and A. Ibarra, Phys. Lett. B **535** (2002) 25 [arXiv:hep-ph/0202239].

[36] K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. **333** (2000) 389.

[37] M. Plumacher, Z. Phys. C **74** (1997) 549. M. Plumacher, Nucl. Phys. B **530** (1998) 207.

[38] K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. D **65** (2002) 043512 [arXiv:hep-ph/0109030].

[39] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169.

[40] M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93** (1995) 879; J. R. Ellis, G. B. Gelmini, J. L. Lopez, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **373** (1992) 399. J. R. Ellis, D. V. Nanopoulos, K. A. Olive and S. J. Rey, Astropart. Phys. **4** (1996) 371; M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B **606** (2001) 518.

[41] W. Buchmuller and D. Wyler, Phys. Lett. B **521** (2001) 291 [arXiv:hep-ph/0108216].

[42] H. Baer, C. Balazs, S. Hesselbach, J. K. Mizukoshi and X. Tata, Phys. Rev. D **63** (2001) 095008 [arXiv:hep-ph/0012205].

[43] J. Ellis, J. Hisano, M. Raidal and Y. Shimizu, arXiv:hep-ph/0206110.

[44] S. Davidson, A. Ibarra, M Plumacher, work in progress.

[45] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B **575** (2000) 61.

[46] J. Ellis and M. Raidal, arXiv:hep-ph/0206174.